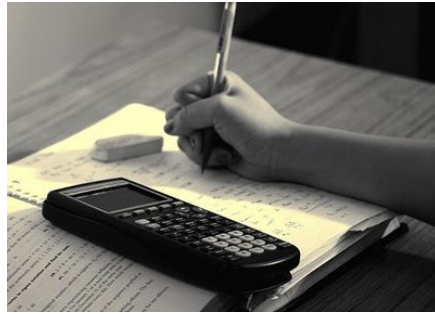


# KATM Bulletin

Kansas Association of Teachers of Mathematics

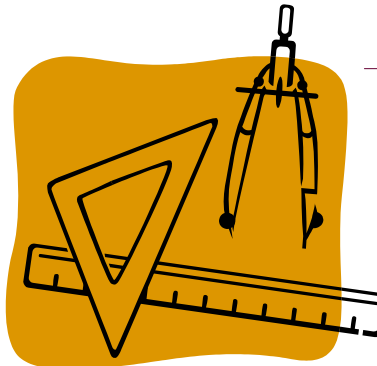


December  
2014

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## Focus Issue: Using Appropriate Tools Strategically



## A Message from our President

The Holidays are fast approaching us. I hope that the first semester has gotten off to a great start. I was excited to see many of you at the KATM Annual Conference in Hays. We enjoyed hearing from our Key-note speaker, Kim Sutton, and the many presenters that shared their tips, tricks, and tools they use everyday in their classrooms across the state. Again, I encourage you to be active in KATM. Your voice is ever so important as we navigate through this transition in math education, more sophisticated testing platforms, and the always changing political landscape here in Kansas. Our next conference will be held in the Fall of 2015 in Wichita, and we look forward to hearing about all of the great teaching going on in Kansas classrooms.

Also this fall, many districts across the state participated in “Break Kite Day” to help not only CETE, but districts as well to have the right tools in place to have a successful testing season in the Spring. KATM is working with our KSDE liaison, Melissa Fast, to get the most up-to-date information for our members and other math teachers across the state. The board looks forward to an update at our January meeting and will quickly get that information out to you.

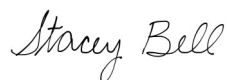
This issue of our Bulletin is focused on the fifth mathematical practice – Use appropriate tools strategically. In thinking about this focus, I ran across this quote from Abraham Maslow. “If the only tool you have is a hammer, you tend to see every problem as a nail.” Are we equipping Kansas’s students with a variety of tools or just giving them a hammer?

I recently heard Bill McCullum speak about the development of the Common Core Standards. He mentioned that he wished that the 8 Mathematical Practice Standards were visually larger than the content standards in the document, as he sees them equally if not more important than the content standards themselves. As teachers, we need to be vigilant in our planning to ensure that students are aware and use these practice standards throughout their mathematical career and their everyday lives.

The 8 Mathematical Practice Standards are in and of themselves tools for solving problems. Teachers have to recognize when to focus on each one throughout their lessons as well as which one makes the most sense at the time. Not all practice standards fit every problem you give students. Nor would you want to focus on all eight at one time necessarily.

Equally so, students have to recognize that they have a math toolbox with these standards in them. They also need to know how to use the practices to solve problems, know when to use them, and did that tool give an answer that makes the most sense? Throughout this issue, you will see lessons or other resources that help teachers guide students through the decision making process of which tool to use and why does it make sense to use it. Students will continue to develop skills at using tools such as rulers, protractors, calculators, graph paper, scales, clocks, number lines, and many other manipulatives that will help them solve problems.

On behalf of the KATM board, we hope that you enjoy this issue of the Bulletin and find it useful. We also want to remind you that we would love to include your submissions for lessons that you have used and want to share with others. Have a Very Happy Holiday Season!



Stacey Bell  
President, KATM  
president@katm.org

## 5 Steps to Use Math Tools Strategically in the Classroom

The very first sentence from CCSS - Mathematical Practice #5 - Use Appropriate Tools Strategically is "Mathematically proficient students consider the available tools when solving a mathematical problem." So how does one go about accomplishing this goal in the classroom?

It is important to realize that we must not force a tool on a student nor expect everyone to use the same tool or to solve the problem in the same way. If we do this, we have proceduralized the use of manipulatives that allows some students to go into automatic pilot. Using tools in the classroom is a balancing act. Given too much guidance takes away the thinking, given too little guidance can cause confusion and possibly never advance the thinking of the student. This is what makes teaching an art versus a science. In this blog I will attempt to describe a procedure to use in the classroom to create that perfect balance.

**Step #1 - Choose the tool(s) to support the lessons' objectives.** Just remember, sometimes students' will use appropriate tools that you will never have thought of using. So be careful that you don't limit the tools. I once saw a student solve a decimal problem using the tens frames. I would have never in my wildest dreams thought to use the tens frames, but it makes a lot of sense in the way he used it to solve the problems.

**Step #2 - Introduce new tools to students.** If it is a tool students have not used before, give free time to explore. Students need opportunity to become familiar with the tools before expected to solve problems using the tool. Once the "free time" has been provided it is important to make certain students understand it is now a math tool in which to solve problems. Ask students to brainstorm how might the tool be used. Record their ideas onto the sign and then hang these tools on a board to remind students of the math tools they have at their disposal.

**Step #3 - Consider the children when planning your lesson.** Are they tactile, visual, etc? Just remember the tools that are effective for one child may not be effective for another. It takes each child different amounts of time to move from the concrete to the representational to the abstract. Time varies from child to child, but it is the ultimate goal to move students through these phases.

**Step #4 - Communication about the use of the tools is at the heart of the effectiveness of using the tools.** Ask appropriate questions frequently. Questions, such as:

"What does the blocks represent?"

"What number did you start with?"

"Why did you put those blocks into those groups?"

"How did you know that three groups of five was 15?"

Students should discuss their work using their own words. This helps them to clarify, perfect, and organize their thinking. Using their own words students should: talk, write, and discuss their mathematical ideas.

**Step #5 - Connect the strategies and the thinking of the students.** The ultimate goal is to move students from concrete to pictorial to abstract (mental math). Students will not always naturally progress without the careful questioning and guidance of teachers. Teachers need to help children to move through these phases.

Math tools do not guarantee success. Sometimes students learn to use manipulatives (tools) in a rote manner. As teachers, we need to reflect on the role of the tools and how they connect to mathematical ideas. It math tools are not the silver bullet, what makes a perfect equation to insure mathematical success? Well here is my idea:

Math Tools + Student Discussion + Engagement + Questioning + Reflection + Cooperative Learning + Perseverance + Exploratory and Deductive Activities = Students Deepened Understanding and Love for Math. My ultimate goal!!

Information courtesy of Math Made Fun, <http://michellef.essdack.org>

## #5 Use appropriate tools strategically

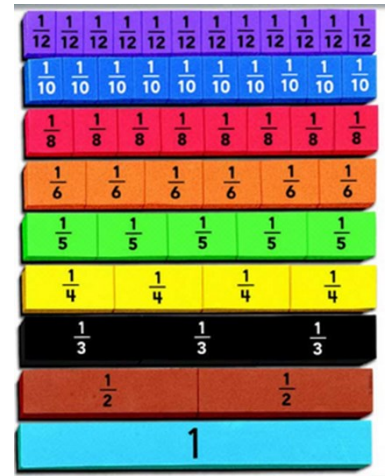
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. (from [corestandards.org](http://corestandards.org))

But what does that look like at different grades????

### K-5

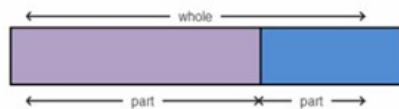
The tools that elementary students might use include physical objects (cubes, geometric shapes, place value manipulatives, fraction bars, etc...), drawings or diagrams (number line, tally marks, tape diagrams, arrays, tables, graphs, etc...), paper and pencil, rulers and other measuring tools, scissors, tracing paper, grid paper, virtual manipulatives, or other available technologies.

Mathematically proficient elementary students choose tools that are relevant and useful to the problem at hand. These include such tools as are mentioned above as well as mathematical tools such as estimation or a particular strategy or algorithm. For example, in order to solve  $3/5 - 1/2$ , a student might recognize that knowledge of equivalent of  $1/2$  is an appropriate tool. Since  $1/2$  is equivalent to  $2/4$ , the result of  $1/2$  of a fifth or  $1/10$ .



This practice is also related to looking for structure (MP.7) which often results in building mathematical tools that can then be used to solve problems.

### Part-Whole Model Addition & Subtraction



$$\text{Part} + \text{Part} = \text{Whole}$$

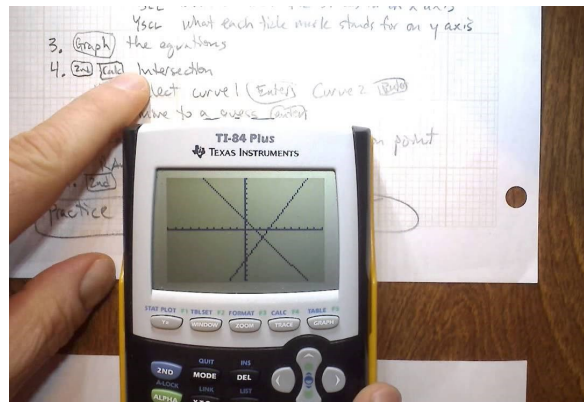
$$\text{Whole} - \text{Part} = \text{Part}$$

## 6-8

In middle school, students might use graphs to model functions, algebra tiles to see how properties of operations apply to algebraic expressions, graphing calculators to solve systems of equations and dynamic geometry software to discover properties of parallelograms.

For example, they might use a spreadsheet simulation to answer the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

A middle school student might use a computer applet demonstrating Archimedes' procedure for approximating the value of pi.



### Supporting PSTs strategic use and integration of visual tools

Chepina Rumsey, Kansas State University

Concrete representations are a tool that students can use to solve or visualize problems, and knowing how to support pre-service teachers' (PST) strategic use and integration of visual tools in their classrooms is an important area to consider. In an elementary mathematics methods class, mathematician Jon Brown and I developed a lesson related to fractions in which we asked the PSTs to solve four fraction questions using four different tools. In order to include a range of fraction representations (linear model, area model, and discrete model), we included Cuisenaire rods, pattern blocks, graph paper, and two-color counters. The PSTs answered each question (for example, show  $1/3$  two ways) using each tool and discussed benefits and limitations of the tools. We strategically chose denominators and questions that promoted discussion among the PSTs. The purpose of the activity was twofold:

- Engage the PSTs in a discussion about the benefits and limitations of the manipulatives for certain types of situations to help them be aware of considerations when they are planning lessons.
- Show the PSTs what it is like when we expose children to a limited set of representations that do not work well for all types of questions they encounter.

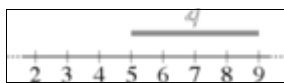
While we used this activity with fractions, similar activities could be included with other domains in mathematics. Exposing PSTs to the various visual tools in mathematics will be helpful as they will soon be teaching children to use many types of tools strategically in their own mathematics classrooms.

## Are tools just manipulatives? Not always!

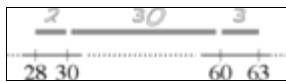
Counters, base-10 blocks, Cuisenaire® Rods, Pattern Blocks, measuring tapes or spoons or cups, and other physical devices are all, if used strategically, of great potential value in the elementary school classroom. They are the “obvious” tools. But this standard also includes “pencil and paper” as a tool, and Mathematical Practice Standard #4 augments “pencil and paper” to distinguish within it “such tools as diagrams, two-way tables, graphs.” The number line and area model of multiplication are two more tools—both diagrammatic representations of mathematical structure—that the CCSS Content Standards explicitly require. So, in the context of elementary mathematics, “use appropriate tools strategically” must be interpreted broadly and sensibly to include many choice options for students.

Essential, and easily overlooked, is the call for students to develop the ability “to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.” This certainly requires that students gain sufficient competence with the tools to recognize the differential power they offer; it also requires that their learning include opportunities to *decide for themselves* which tool serves them best. It also requires curricula and teaching to include the kinds of problems that genuinely favor different tools. It may also require that, from time to time, a particular tool is prescribed—or proscribed—until students develop a competency that would allow them to make “sound decisions” about which tool to use.

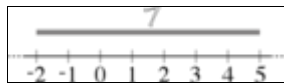
The number line is sometimes regarded just as a visual aid for children. It is, in fact, a sophisticated image used even by mathematicians. For young children, it helps develop early mental images of addition and subtraction that connect arithmetic with measurement. Rulers are just number lines built to spec! This number line im-



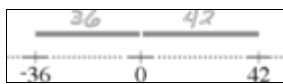
age shows “the distance from 5 to 9” or “how much greater 9 is than 5.” Children who see subtraction that way can use this model to see “the distance between 28 and 63” as



35, and to do so without crossing out digits and borrowing and following a rule they may only barely understand. In fact, many can learn to see this model in their heads, too, and do this subtraction mentally. This is essentially how clerks used to “count up” to make change. The number line model also extends naturally to decimals and fractions by “zooming in” to get a more detailed view of that line between the whole numbers. And it extends equally naturally to negative numbers. It thereby unifies arithmetic, making sense of what is otherwise often seen as a collection of independent and hard-to-remember rules. We can *see* that the distance from -2 to 5 is






the number we must add to -2 to get 5: . And we can *see* why  $42 - (-36)$  can also be writ-

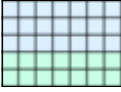


ten as  $42 + 36$ : the “distance from -36 to 42 is . The number line remains useful as students study data, graphing, and algebra: two number lines, at right angles to each other, label the addresses of points on the coordinate plane.

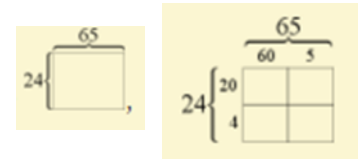
The area model of multiplication is another powerful tool that lasts from early grades through college mathematics

Images like  along with appropriate questions like “how many columns, how many rows, how many little squares” help establish the small multiplication facts. So might pure drill, of course, but this array image goes much further. Seeing the

same array held in different positions like  and  makes clear that we can label any of these  $3 \times 4$  or  $4 \times 3$  and

the number of little squares is always 12. In grades 3 through 5, array pictures like  help clarify the distributive property of multiplication. This is the property that makes multi-digit multiplication possible, makes sense of the standard multiplication and division algorithms, and underlies the multiplication that students will encounter in algebra. In this picture, we *see* that “two 7s plus three 7s is five 7s.” The conventional notation of the idea,  $2 \times 7 + 3 \times 7 = (2 + 3) \times 7$ , can be a useful, even informative, summary *after* students already understand: a conclusion, rather than a starting place.

A schematic version of this image—the area model of multiplication—organizes students’ thinking as they learn multi-digit multiplication. In the  $3 \times 4$  array, counting the squares was not impractical, though remembering the fact was certainly more convenient. But in a  $65 \times 24$  array,



neither a memorized fact nor counting are practical. Instead, by partitioning the array, we get a set of steps for which a combination of memorized facts and an understanding of place value help.

This image, combined with a spreadsheet-like summary, making total sense of what can otherwise feel like

		65			
		60	5		
24	{	20	1200	100	1300
		4	240	20	260
			1440	120	

mary, models the conventional algorithm exactly—an arbitrary set of steps.

The same image also allows students to acquire and understand the algorithm for division as a process of “undoing” multiplication, greatly simplifying the learning of a part of arithmetic that has a long history of being difficult. What makes this a powerful tool is that it serves the immediate goals of elementary school arithmetic in a way that prepares students for algebra.

Algebraic multiplication of  $(x + 5)(b + 4)$  which has no “carry” step, is modeled perfectly by exactly the same tool. And for  $147 \times 46$  or for  $(y + s - 5)(q - 2)$ , no new image is needed. We just extend the “area” model to include the extra terms.

These versatile tools build mental models that last. What makes a tool like the number line or area model truly powerful is that it is not just a special-purpose trick or temporary crutch, but is faithful to the mathematics and is extensible and applicable to many domains. These tools help students make sense of the mathematics; that’s why they last. And that is also why the CCSS mandates them.

Information from [thinkmath.edc.org](http://thinkmath.edc.org), courtesy of Educational Development Center

## Implementing Standards for Mathematical Practices

### #5 Use appropriate tools strategically.

#### Questions to Develop Mathematical Thinking

- What mathematical tools could we use to visualize and represent the situation?
- What information do you have?
- What do you know that is not stated in the problem?
- What approach are you considering trying first?
- What estimate did you make for the solution?
- In this situation, would it be helpful to use a (graph, number line, ruler, diagram, calculator, manipulative, etc.)?
- Why was it helpful to use...?
- What can using a \_\_\_\_\_ show us that \_\_\_\_\_ may not?
- In what situations might it be more informative or helpful to use...?

#### Implementation Characteristics: What does it look like in planning and delivery?

Task: elements to keep in mind when determining learning experiences

Teacher: actions that further the development of math practices within their students

##### Task:

- Lends itself to multiple learning tools. (Tools may include; concrete models, measurement tools, graphs, diagrams, spreadsheets, statistical software, etc...)
- Require students to determine and use appropriate tools to solve problems
- Ask students to estimate in a variety of situations:
  - \*a task when there is no need to have an exact answer
  - \*a task when there is not enough information to get an exact answer
  - \* a task to check if the answer from a calculation is reasonable

##### Teacher:

- Demonstrates and provides students experiences with the use of various math tools. A variety of tools are within the environment and readily available.
- Questions students as to why they chose the tools they used to solve the problem.
- Consistently models how and when to estimate effectively, and requires students to use estimation strategies in a variety of situations.
- Asks students to explain their mathematical thinking with the chosen tool.
- Ask students to explore other options when some tools are not available.



# How can I encourage practice 5 in my classroom?

## Student Actions

- Choose from a variety of mathematical tools that are readily available (unifix cubes, base ten blocks, number lines, tables, etc.)
- Choose a tool that will help them best solve the problem and know certain tools will not be helpful.
- Use technological tools and understands the effects and limitations of those tools.
- Use outside resources to help them solve the problem.
- Use technological tools to explore and deepen their understanding of concepts.
- Change the tool if the tool does not help reach an accurate solution.

## Teacher Actions

- Provide students with the experience with a variety of tools.
- Facilitate discussion regarding the appropriateness of different tools.
- Allow students to choose their choice of tools and think outside the box.
- Use anchor charts when a new tool is used and when it is used in a different way,
- Use virtual manipulatives and other technology tools in the classroom.
- Allow students time to explore these virtual tools.
- As students solve problems, roam and ask for explanation on how the tools are being used.
- Choose students who used different tools to share with the class. Facilitate the discussion.

## Open-Ended Questions

- How might you represent the problem using your tool choice?
- How is this tool helping you to understand and solve the problem?
- What tools have we used (number line, function table, etc.) that might help you to organize the information from the problem?
- What might be another tool that may help clarify the problem and help you to solve the problem?
- How is this tool/strategy helping you to solve the problem? What else might you try?
- How did the (function table) help \_\_\_\_\_ to solve the problem?
- How might a picture or math tool help you solve the problem?

Using the tool of “decomposition” to think flexibly about numbers

by Liz Peyser, VP for Middle Schools

There is a thread in the K-12 standards of using “decomposition” to break apart numbers as a computation strategy combined with visual models as tools. This thread begins in Kindergarten, is developed with computation of multi-digit numbers and fractions, and is applied to abstract numbers in high school mathematics. The concept of decomposition is also used in geometric measurements of angles, area, and volume.

To begin, please find the solution to this problem **mentally**. [Do not use paper, pencil, calculators or other devices!]:

*Find the total cost of 3.5 pounds of apples at \$4.50 per pound...*

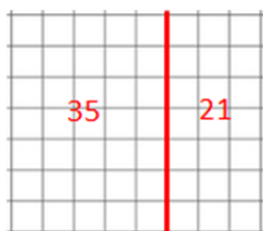
After you have found the answer mentally, write down what you did to find the total cost.

Now, grab some cubes! We will begin to model decomposition with cubes using standard K.OA.3. Take 7 cubes and model the 7 cubes as  $3 + 4$ , or as  $6 + 1$ . What would be another way to show this quantity?  $5 + 2$ ? Students begin to see quantities as “decomposed” versions of that quantity.

In standards 1.OA.6, 1.NBT.2b, and 2.OA.2 this decomposition is applied to larger quantities and combined with strategies to make “friendlier” numbers. For example, take 13 cubes and decompose them to make  $10 + 3$ . Students can think about “Making a Ten” to create simpler problems. How could we model  $8 + 6$ ? Take 2 cubes from the 6, move them to the 8 to make 10. The new math model is  $10 + 4$ . We could also use a “Doubles + 1” strategy to decompose  $6 + 7$ . Model this with the cubes to show  $6 + 6 + 1$ .

In standards 2.OA.4, and 3.OA.3 students use the idea of groups to model multiplication. If there are 15 cubes, we can decompose this into 3 groups of 5. We could arrange them in an array or area model to show 3 rows of 5 cubes. With two seemingly unrelated standards, 3.OA.5 and 3.MD.7c, 3<sup>rd</sup> grade students connect the “area” model of area measurement with decomposition to multiply larger numbers. What is the quantity of  $7 \times 8$ ? If a student can’t remember this fact, use  $7 \times 5 + 7 \times 3$ .

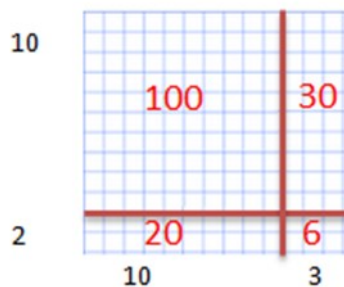
Shown here with an area model:  $7 \times 5$        $7 \times 3$



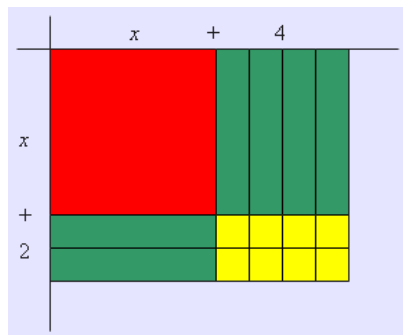
Two number models using the distributive property are  $7(5 + 3)$  and  $7(5) + 7(3)$ .

Try this using  $3 \times 14$ . Decompose 14 into  $10 + 4$  and write two number models to represent the product. How are your number models related to the standard algorithm for multiplication? (Where in the algorithm are three groups of 4 ones and 3 groups of 10).

In 4<sup>th</sup> grade 4.NBT.5, students use decomposition with the array tool to multiply two 2-digit numbers. Draw  $12 \times 13$  on grid paper and show the partial products. How are the partial products related to the standard algorithm?



This standard leads directly to application in secondary math, where the number models for this would be  $(10 + 2)(10 + 3)$  and students use the distributive property to find the sum of four products. In high school Algebra, this is often shown with algebra tiles and the number model  $(x + 2)(x + 4)$  to show the product of two binomials.



The decomposition tool is also used with fractions in 5<sup>th</sup> grade to solve the problem of  $3\frac{1}{2} \times 4\frac{1}{2}$ . How can we use decomposition to find partial products? Think of these numbers as  $(3 + \frac{1}{2}) \times (4 + \frac{1}{2})$ . Could you use the distributive property to find the sum of four products? What is the final answer?

How is this related to your answer to the apple problem?

*[Answer: 3 groups of 4, 3 groups of  $\frac{1}{2}$ ,  $\frac{1}{2}$  groups of 4 and  $\frac{1}{2}$  groups of  $\frac{1}{2}$  is equal to  $12 + 1\frac{1}{2} + 2 + \frac{1}{4}$ , or  $15\frac{3}{4}$ ]*

## High School Lesson: Modeling: Having Kittens

### MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Interpret a situation and represent the constraints and variables mathematically.
- Select appropriate mathematical methods to use.
- Make sensible estimates and assumptions.
- Investigate an exponentially increasing sequence.
- Communicate their reasoning clearly.

### MATERIALS REQUIRED

- Each individual student will need a calculator, a copy of the assessment task Having Kittens, the questionnaire How Did You Work, a mini-whiteboard, an eraser, and a pen.
- Each small group will need a copy of all the Sample Responses to Discuss, a large sheet of paper for making a poster, and felt-tipped pens. Graph paper should be kept in reserve and used only when requested.
- There are some projector resources to help you with whole-class discussions. Spreadsheet software might also be helpful if available.

### TIME NEEDED

20 minutes before the lesson, a 90-minute lesson, and 10 minutes in a follow-up lesson (or for homework). Timings given are only approximate.

### SUGGESTED LESSON OUTLINE

#### **Introduction: Cats and Kittens (10 minutes)**

#### **Collaborative activity: Producing a joint solution (20 minutes)**

Organize the class into small groups of two or three students and give each group a large sheet of paper and a felt-tipped pen. Ask students to try the task, combining their ideas.

#### **Checking posters (10 minutes)**

After students have had sufficient time to attempt the problem, ask one student from each group to visit another group's poster.

#### **Sharing different approaches (20 minutes)**

#### **Whole-class discussion: Comparing different approaches (20 minutes)**

This lesson is from the Mathematics Assessment Project website, and is related to math practice #5, use appropriate tools strategically. Only portions of the available materials have been reprinted in this Bulletin. For all resources, visit the website: <http://map.mathshell.org.uk/materials>

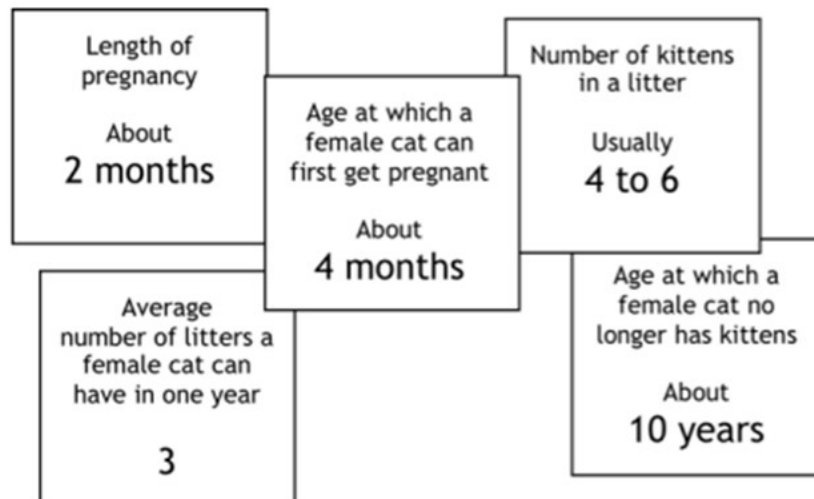
## Having Kittens

Here is a poster published by an organization that looks after stray cats.



Figure out whether this number of descendants is realistic.

Here are some facts that you will need:



## Modeling Kittens Lesson, continued

### How Did You Work?

Check (✓) the boxes and complete the sentences that apply to your work.

1. Check (✓) the facts you used:

Length of pregnancy	<input type="checkbox"/>
About 2 months	

Age at which a female cat can first get pregnant	<input type="checkbox"/>
About 4 months	

Number of kittens in a litter	<input type="checkbox"/>
Usually 4 to 6	

Average number of litters a female cat can have in one year	<input type="checkbox"/>
3	

Age at which a female cat no longer has kittens	<input type="checkbox"/>
About 10 years	

2. Our group work was better than my own work

Our joint solution was better because .....

.....

.....

.....

3. We made some assumptions

These assumptions were .....

.....

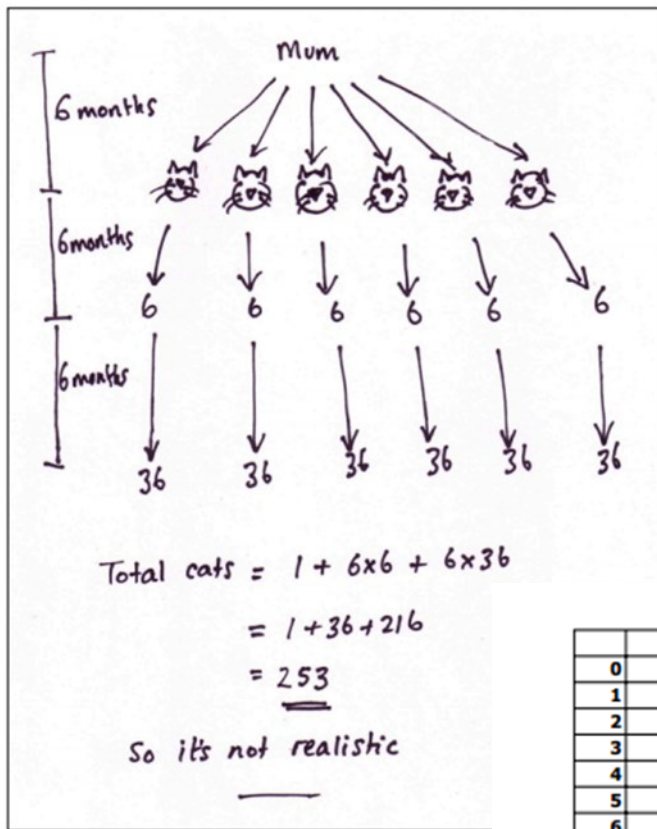
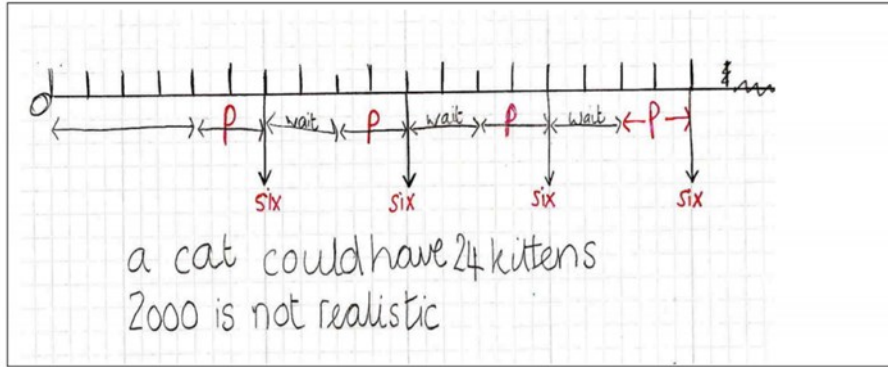
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4. Another group has understood my poster

Modeling Kittens Lesson, continued

Sample Student Work:



Generation of kittens

	0	1	2	2	2	2	3	3	3	4	Cum. total
0	1	6									7
1											7
2											7
3											7
4		6									13
5											13
6			36								49
7											49
8		6									55
9											55
10			36	36							127
11											127
12		6					216				349
13											349
14			36	36	36						457
15											457
16		6					216	216	216		1111
17											1111
18			36	36	36	36				1296	2551

Time in months

## Modeling the Distributive Property, a Middle School Lesson

Contributed by Lori Stockstill, Hamilton MS

Objective: Students will find the pattern involved from creating increasing groups of the same expression and how the quantity can be expressed in two different formats, leading to the understanding of the distributive property.

CCSS domain connections: Expressions/Equations (building from K-5 work with multiplication in 3.OA.1 and 3.OA.5)

Standards for Mathematical Practice connections: #2, #3, #4, #5, #7, #8

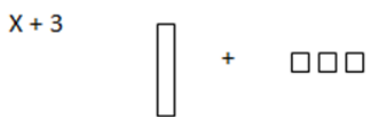
**Materials:**

Algebra tiles, Whiteboards (optional) or paper

**Opening:**

Introduce the students to the algebra tiles, where the long bar is equal to “x” and the smaller unit squares represent constants. The students can use the red side of the tiles to represent negative numbers. (If algebra tiles are not accessible, students can use two different manipulatives – one for “x” and one for the constant.

**Worktime #1:** Have the students model and draw these expressions on their whiteboards (or paper):



**Probing question:** Is there another way to model  $2x - 4$  using the red tiles?

**Worktime #2:**

Have students model  $x + 5$  with the tiles.

Have students show two groups of  $x + 5$ .



Have students present their representations. Discuss the different representations. Some probing questions might be: Is the quantity the same? How do you know? How would you write two groups of  $x + 5$ ? Answer:  $2(x + 5)$

How would you say that? Answer: 2 times the quantity of  $x + 5$

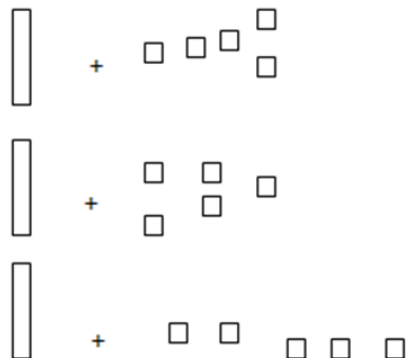
What is the other expression for the same quantity? Answer:  $2x + 10$

What is happening that makes the two expressions refer to the same quantity?



**Worktime #3:**

Pose the question: "What would it look like if we added another group?"



What are two ways to write this? *Answers:  $3(x + 5)$  and  $3x + 15$ .*

**Closing:**

What if we added another group? What would that look like?

Ask if the students could add " $x + 2$ "? Why not? (*Answer: the groups are not the same*)

How can you explain what is always happening when we have multiples of the same group and what are the two ways you can write that?

End with the critical question: Does  $5(2x-3)$  equal  $10x - 3$  or  $10x - 15$ . Explain your answer.

Have groups create explanations (possibly on whiteboards for a Carousel feedback. Students could post sticky notes).

Can you create a rule to relate the two expressions that will work every time?

*Answer: the coefficient is multiplying all of the quantities on the inside of the parentheses by that same factor.  $3(2 + x) = 6 + 3x$*

**Checking for understanding using your rule:**

What if the quantity was  $10x + 30$ ? Is there another way to write that? How do you know the two expressions are equal?

*Answer:  $10(x + 3)$ . They are the same because 10 groups of  $x + 3$  would create 10 groups of  $x$  and 10 groups of 3 (or 30).*

How we would say  $5(2x-3)$ ?

*Answer: "5 times the quantity of  $2x - 3$ "*

Middle School lesson, continued

1. Draw a diagram here and on your whiteboard for  $x + 5$ :
2. Show two groups of  $x + 5$  here and on your whiteboard:
3. Two diagrams and two numerical expressions to represent 3 groups of  $x + 5$  are...
4. What is happening that makes the two expressions refer to the same quantity?
5. Does  $5(2x + 3)$  equal  $10 + 3$  or  $10x + 15$ ? Explain.
6. A rule that will work all the time for the Distributive Property is....

## Performance Assessment Task Grade 2: Carol's Numbers

The task challenges a student to demonstrate understanding of concepts involved in place value. A student must understand the relative magnitude of whole numbers from the quantity of a digit in a particular place in the number and use this understanding to compare different numbers. The student must make sense of the concepts of sequences, quantity, and the relative position of numbers.

### Common Core State Standards Math - Content Standards

Number and Operations in Base Ten

Understand place value.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

- 100 can be thought of as a bundle of ten tens — called a “hundred.”
- The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundred s (and 0 tens and 0 ones)

2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using  $>$ ,  $=$  or  $<$  symbols to record the results of comparisons.

### Common Core State Standards Math – Standards of Mathematical Practice

MP.5 Use appropriate tools strategically, MP.7 Look for and make use of structure

## Carol's Numbers

Carol has three number cards.



1. What is the largest three-digit number Carol can make with her cards?

--	--	--

2. What is the smallest three-digit number Carol can make with her cards?

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## KATM Bulletin

Explain to Carol how she can make the smallest possible number using her three cards.

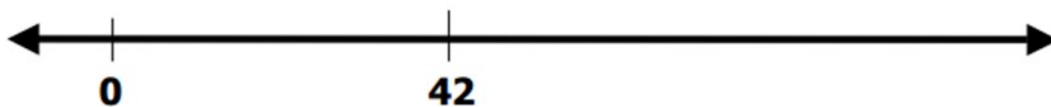
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Carol's teacher drew a number line on the board.



3. About where would 85 be? Place 85 on the number line where it belongs.
4. About where would 21 be? Place 21 on the number line where it belongs.
5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

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Lesson courtesy of [insidemathematics.org](http://insidemathematics.org) Additional resources related to this lesson, including grading rubrics, student work samples, common misunderstandings and teaching implications can be found at :

<http://www.insidemathematics.org/assets/common-core-math-tasks/carol's%20numbers.pdf>

Do you like what you find in this Bulletin? Would you like to receive more Bulletins, as well as other benefits?

Consider becoming a member of KATM.

For just \$15 a year, you can become a member of KATM and have the Bulletin e-mailed to you as soon as it becomes available. KATM publishes 4 Bulletins a year. In addition, as a KATM member, you can apply for two different \$1000 scholarship.

Join us today!!! Complete the form below and send it with your check payable to

KATM to:  
 Margie Hill  
 KATM-Membership  
 15735 Antioch Road  
 Overland Park, Kansas 66221

Name \_\_\_\_\_

Address \_\_\_\_\_

City \_\_\_\_\_

State \_\_\_\_\_

Zip \_\_\_\_\_

Home Phone \_\_\_\_\_

HOME or PERSONAL EMAIL:

\_\_\_\_\_

Are you a member of NCTM? Yes \_\_\_ No \_\_\_

Position: (Check only one)

- Parent
- Teacher:: Level(s) \_\_\_\_\_
- Dept. Chair
- Supervisor
- Other

\_\_\_\_\_

KANSAS ASSOCIATION MEMBERSHIPS

- Individual Membership: \$15/yr. \_\_\_\_
- Three Years: \$40 \_\_\_\_
- Student Membership: \$ 5/yr. \_\_\_\_
- Institutional Membership: \$25/yr. \_\_\_\_
- Retired Teacher Membership: \$ 5/yr. \_\_\_\_
- First Year Teacher Membership:\$5/yr. \_\_\_\_
- Spousal Membership: \$ 5/yr. \_\_\_\_
- (open to spouses of current members who hold a regular Individual Membership in KATM)

Greetings Kansas Math Teachers!

I hope that you find this Bulletin full of useful information about how to apply math practice #5 in your classroom, with your students. We look forward to continuing to explore other math practices in future issues. We are trying to include as much useful information as possible, and we would love to include your submission as well! Please contact me if you would like to submit to our Bulletin.

Sincerely,

Jenny Wilcox, KATM Bulletin Editor

*Jenny Wilcox*

## Highlighted Resource Review

Review by Lisa Lajoie-Smith

The Mathematics Assessment Project website has focused lessons for Grade 6 thru High School that embed the Mathematical Practices within those lessons as well as which state standard is being addressed within the lesson. Each lesson is all inclusive with a suggested outline of the structures and time for each section. Each lesson has sample student responses for teachers and students to have as a point of reference. There are approximately 200 lessons available on this site for use. You can sort the lessons by task type, standards or mathematical practices. There are 5 professional development modules for teachers as well.

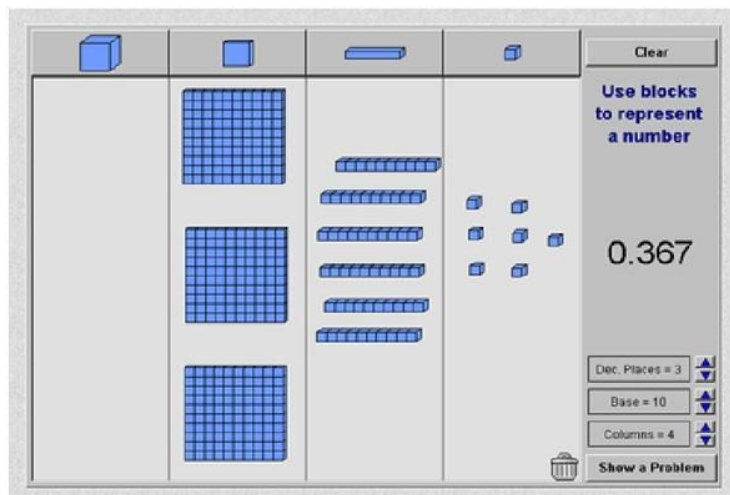
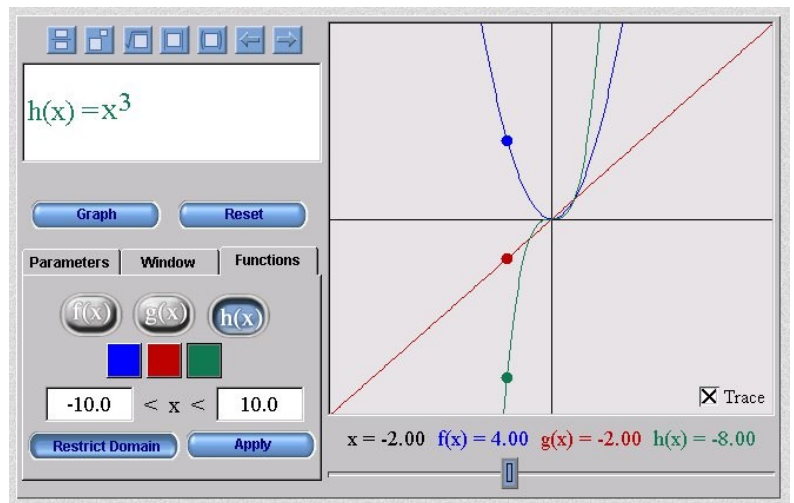
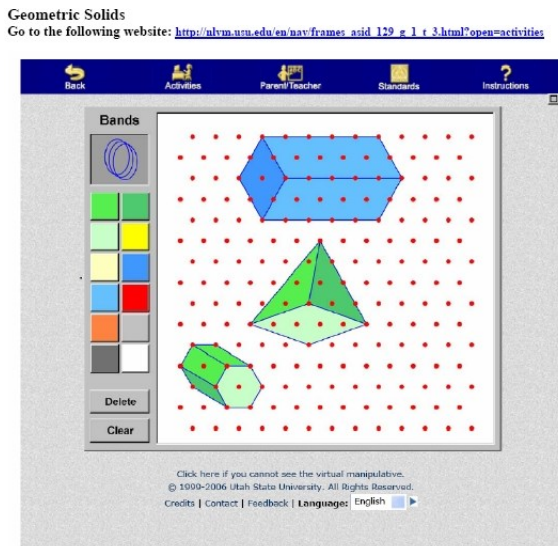
The screenshot shows the Mathematics Assessment Project website interface. At the top, it says "Mathematics Assessment Project COMMON CORE STATE STANDARDS Mathematical Practices for All Grades". Below this is a navigation bar with links for Home, MAP Overview, Lessons, Tasks, Items, Professional Development, 7th Math Exam, Standards, Instructions, and Log In. There is a search bar with "Formative Assessment Lessons (480)", "Mathematical Practices", "All Grades", and "Find" buttons. Below the search bar, there are several sections: "Found 2 matches", "All Grades", "Standards for Mathematical Practice", "Mathematical Content (High School)", and "Mathematical Content (Middle School)". The main content area displays a list of lessons under the heading "MP1: Make sense of problems and persevere in solving them". The lessons include titles like "Optimization Problems: Broomcans", "Maximizing Area: Gold Rush", "Optimizing Security Cameras", "A Mixture of Slope", "Calculating Volume of Composed Objects", "Using Dimensions: Catering a Sports Day", "Proportional Relationships: Finding Student Scores: The Proportional Problem", "Cutting Steaks Using Probability", "Estimating Counting Tapes", "Evan, Maden, Mike, and Renge", "Solving Real-Use Problems: Selling", "Geometry Problems: Circles and Triangles", "Equations of Circles 2", "Sharing Costs: Traveling to School", "Testing a New Product", "Finding Equations of Parallel and Perpendicular Lines", "Solving Geometry Problems", "Identifying Similar Triangles", "Comparing Data", "Inscribing and Circumscribing Polygons", "Functions and Everyday Situations", "Transforming 3D Figures", "Generalizing Patterns: The Influence of Two Squares", "Importance of Inductive and Deductive Reasoning", "Representing Data Using Grouped Frequency Graphs and Box Plots", "Modeling Conditional Probabilities 4", "Solving Real-Use Problems: Baseball", "Applying Properties of Exponents", "Investigating Composite Solids", "The Pythagorean Theorem: Square Areas", "Medical Testing", "Quadratic Quadratics", "Factors and Multiples", "Modeling Honey Bees", "Solving Linear Equations in One Variable", "Using Coordinates to Measure and Represent Data", "Understanding Reverse Engineering", "Evaluating Statements About Number Operations", "Mathematical Content (High School)", "Modeling: Making Predictions About Luck: A Lucky Spin", "Using Equations and Inequalities", "Possible Triangle Constructions", "Modeling: Hot or Cold", "Mathematical Content (Middle School)", "Classifying Solutions to Systems of Equations", "Probability Games", "The Area of a Circle", "Probability Games", "The Area of a Circle", "Probability Games", "The Area of a Circle".

<http://map.mathshell.org.uk/materials/index.php>

# Highlighted Resource Review

Review by Jenny Wilcox

The National Library of Virtual Manipulatives is a resource that offers a wide-variety of online manipulatives and activities. The activities are sortable by grade band (PreK-2, 3-5, 6-8 and High School) or by strand (Number and Operations, Algebra, Geometry, Measurement, Data Analysis and Probability). The activities include virtual versions of familiar manipulatives such as Base 10 Blocks, Algebra Tiles, Pattern Blocks, number lines, fraction bars, percent grids, spinners, algebra balance scales and geoboards. There are also several other activities and games such as Mastermind, Tower of Hanoi, Fifteen Puzzle and Triominoes. As we consider how to use appropriate tools strategically, this website may offer more access to a wide variety of online tools for our students.



# *In the coming issues— update!!!*

*This issue is the first in a series of bulletins that will be highlighting the Standards for Mathematical Practice. This is where our series will be heading in future Bulletins.*

◆ *February 2015 Bulletin will focus on #4, **Model with mathematics***

Through this practice, mathematically proficient students create models to represent the relationships among the quantities in a situation. From the models, they gain insight into how to solve the problem, or what pattern they notice so they can create a new model. This is an exciting Practice that really makes the mathematics “visual” for students by using diagrams, tables, graphs, flowcharts, equations and formulas. Students can ask themselves these questions:

“Can I make my life easier by using math to optimize solutions?”

“Is there a way I can demonstrate real world problems using math?”

“Could a mathematical model make the problem situation and potential solution clearer?”

(courtesy of Weber State University)

◆ *April 2015 Bulletin will focus on #3, **Construct viable arguments and critique the reasoning of others.***

Teachers who are developing students’ capacity to “construct viable arguments and critique the reasoning of others” require their students to engage in active mathematical discourse. This might involve having students explain and discuss their thinking processes aloud, or signaling agreement/disagreement with a hand signal. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. A middle childhood teacher might post multiple approaches to a problem and ask students to identify plausible rationales for each approach. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They justify their conclusions, communicate them to others, and respond to the arguments of others. (description courtesy of insidemathematics.org)



## CALL FOR SUBMISSIONS

### Your chance to publish and share your best ideas!

The KATM Bulletin needs submissions from K-12 teachers highlighting the mathematical practices listed above. Submissions could be any of the following:

- ◇ Lesson plans
- ◇ Classroom management tips
- ◇ Books reviews
- ◇ Classroom games
- ◇ Reviews of recently adopted resources
- ◇ Good problems for classroom use

Email your submissions to our Bulletin editor: [wilcojen@usd437.net](mailto:wilcojen@usd437.net)

Acceptable formats for submissions: Microsoft Word document, Google doc, or PDF.

'Leaders... will be explorers, adventurers, trailblazers... leaders of leaders... They will gather around them people who have the future in their bones.' **Rowan Gibson** 'Re Thinking the Future

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## BE A LEADER

---

as a Kansas math teacher!

## Capitol Federal Mathematics Teaching Enhancement Scholarship

Capitol Federal Savings and the Kansas Association of Teachers of Mathematics (KATM) have established a scholarship to be awarded to a practicing Kansas (K-12) teacher for the best mathematics teaching enhancement proposal. The scholarship is \$1000 to be awarded at the annual KATM conference. The scholarship is competitive with the winning proposal determined by the Executive Council of KATM.

### PROPOSAL GUIDELINES:

The winning proposal will be the best plan submitted involving the enhancement of mathematics teaching. Proposals may include, but are not limited to, continuing mathematics education, conference or workshop attendance, or any other improvement of mathematics teaching opportunity. The 1-2 page typed proposal should include

- A complete description of the mathematics teaching opportunity you plan to embark upon.
- An outline of how the funds will be used.

An explanation of how this opportunity will enhance your teaching of mathematics.

### REQUIREMENTS:

The successful applicant will meet the following criteria:

- Have a continuing contract for the next school year in a Kansas school.
- Teach mathematics during the current year.

Be present to accept the award at the annual KATM Conference.

### APPLICATION:

To be considered for this scholarship, the applicant needs to submit the following no later than **June 1 of the current year**.

- A 1-2 page proposal as described above.

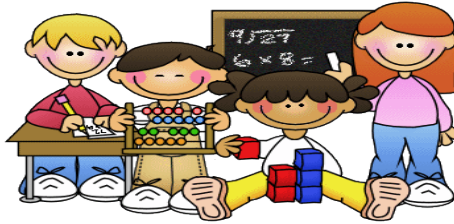
Two letters of recommendation, one from an administrator and one from a teaching colleague.

### PLEASE SUBMIT MATERIALS TO:

Betsy Wiens, Phone: (785) 862-9433, 2201 SE 53<sup>rd</sup> Street, Topeka, Kansas, 66609

Don't be afraid to try...go for it!  
Submit a proposal for whatever  
cool thing you've been hoping to  
do in your classroom!

## KATM Cecile Beougher Scholarship ONLY FOR ELEMENTARY TEACHERS!!



A scholarship in memory of Cecile Beougher will be awarded to a practicing Kansas elementary (K-6) teacher for professional development in mathematics, mathematics education, and/or mathematics materials needed in the classroom. This could include attendance at a local, regional, national, state, or online conference/workshop; enrollment fees for course work, and/or math related classroom materials/supplies.

The value of the scholarship upon selection is up to \$1000:

- To defray the costs of registration fees, substitute costs, tuition, books etc.,
- For reimbursement of purchase of mathematics materials/supplies for the classroom

**An itemized request for funds is required. (for clarity)**

### ***REQUIREMENTS:***

The successful candidate will meet the following criteria:

- Have a continuing contract for the next school year as a practicing Kansas elementary (K-6) teacher.
- Current member of KATM (if you are not a member, you may join by going to [www.katm.org](http://www.katm.org). The cost of a one-year membership is \$15)

### ***APPLICATION:***

To be considered for this scholarship, the applicant needs to submit the following no later than June 1 of the current year:

1. A letter from the applicant addressing the following: a reflection on how the conference, workshop, or course will help your teaching, being specific about the when and what of the session, and how you plan to promote mathematics in the future.
2. Two letters of recommendation/support (one from an administrator and one from a colleague).
3. A budget outline of how the scholarship money will be spent.

Notification of status of the scholarship will be made by July 15 of the current year. Please plan to attend the KATM annual conference to receive your scholarship. Also, please plan to participate in the conference.

### ***SUBMIT MATERIALS TO:***

Betsy Wiens  
2201 SE 53<sup>rd</sup> Street  
Topeka, Kansas 66609

*Go to [www.katm.org](http://www.katm.org) for more guidance on this scholarship*





## KLFA

### KLFA Focuses on Individual Plans of Study and College and Career Readiness

•October 16, 2014•

Kansas Learning First Alliance (KLFA) met October 16 at the Kansas Association of School Boards (KASB) building. Approximately 25 educators were in attendance.

Constitutional Resolutions were moved and passed to: a) remove the constitutional ban that limits a chair to two years of service for the 2015-2016 only, b) remove the requirement of a vice chairman for 2014-2015 only, and c) reimburse the chair for travel and administrative expenses for 2014-2015 and 2015-2016 only. Dayna Richardson, KLFA Chair, will serve a third year.

Angie McDonald, McPherson #USD 418 Director of Instruction, shared the McPherson school district's **Individual Plan of Study**. Their plan includes:

Citizen: Character, service, and activity involvement

College: Academic readiness, academic rigor, and academic behaviors

Career: Work Keys, attendance, drug testing for students in activities, career pathways

McPherson employed three CCR Advocates at the high school and one at the middle school to work with counselors and meet with students and families two times each year starting in the 6<sup>th</sup> grade to discuss the Individual Plan of Study. Board of Education members have had positive comments and the preliminary data is looking good. Weekly meetings are held to analyze nine measures of each child. High school counselors' focus has changed to support social and emotional awareness and stability.

A legislative/political update was given by Tom Krebs (KASB), Mark Farr (KNEA), Claudette Johns (KNEA), Cheryl Semmel (USA), Deena Burnett (AFTKS), and Kathy Busch (KSBE). All members stressed the importance of being an informed voter, of voting and voting early.

Continuing with the KLFA theme of professional learning, Dayna Richardson shared copies of the new **Kansas Education Systems Accreditation (KESA)** rubrics. Professional learning is infused in each of the 5Rs. If professional learning isn't the answer, what is?

KLFA is focusing on six strands throughout this year – Standards, State Assessments, Individual Plans of Study, Educator Evaluation Systems, Rose Capacities, and Accreditation. During the work session, KLFA members chose a strand and began to brainstorm ways to promote the strand throughout Kansas.

## KATM Board Meeting Sunday, October 12, 2014

Welcome to new board members/positions – Jaclyn Pfizenmaier: Zone 2 Coordinator,  
Whitney Czajkowski-Farrell: Zone 3 Coordinator

### **Possible solutions for increasing membership (Goal of 400):**

- Offer membership as part of conference registration.*
- Make membership an automatic part of conference registration*
- Student membership is \$5. Promote this!*
- Discount on conference registration for active members.*
- Offer Vendor bucks at the conference.*
- Ask teachers why they join KATM; ask why they don't.*

**At the January meeting, we will come prepared to discuss membership, make decisions, and implement.**

*Betsy has requested a scholarship from Cap Fed for 2015. Stacey will present the Cecile Beougher and Cap Fed Scholarships during the KATM Opening Session. Stacey and Janet will present the Ray Kurtz Award.*

*Membership – From mid-August to late September, 2014, Zone Coordinators: Lisa, Jaclyn, and Jeannett, along with Margie and Betsy emailed over 800 members with reminders to renew their membership. If we purge all non-renewed members, we are down to 105 active members.*

Finance – David Fernkopf  
Treasurer's Report – Current Balance - \$19,077.50

Publications – Jenny Wilcox

- Themed issues – The Standards for Mathematical Practice*
- October – Eight Standards for Mathematical Practice (in general)*
- December – Standard 5 (Use appropriate tools strategically.)*
- February – Standards 4 and 3 (Model with mathematics; Construct viable arguments and critique the reasoning of others.)*

Featured Websites Discussion – Jenny asked Board members to submit websites along with a short summary/description. KSDE has an Extended Data Review Tool in order to evaluate a website. The Publications Committee will share this at the January meeting. **Board members are invited to submit website ideas to Jenny.**

2015 Conference Update – Stacey Bell

- Location – Wichita (either WSU or Friends University)
- Date – TBD (Check the KATM website for updates.)

Zones Report Out

Any potential events during this school year?

Zone 2 – Melisa reported that many districts in Kansas are using Engage New York Math. She will be hosting an evening meeting for teachers and parents. Melisa will draft a plan to submit for funding.

Jeannett suggested a Zone Mentor.

Jenny suggested that the conference presenters be asked to present the same session in the Zone. Janet will send Jenny the presenter spreadsheet.

# KATM Executive Board Members

**President:** Stacey Bell  
Instructional Coach, Shawnee Heights Middle School  
bells at usd450.net, 785-379-5830



**Past President, NCTM Rep:** Melisa Hancock  
Consultant, Kansas State University  
melisa at ksu.edu



**Secretary:** Janet Stramel  
Assistant Professor, Fort Hays State Univ.  
jkstramel at fhsu.edu



**Membership Co-chairs:** Margie Hill  
Instructor, Kansas University  
785-864-0554  
marghill at @ ku.edu



**Membership Co-Chair:** Betsy Wiens  
Math Consultant  
albf2201 at aol.com



**Treasurer:** David Fernkopf  
Principal, Overbrook Attendance Center, dferkopf at usd434.us



**KSDE Liaison:** Melissa Fast  
Math Education Consultant  
900 SW Jackson St, Suite 663  
Topeka, KS 66612  
785-296-3486  
mfast at ksde.org



**President Elect:** Pat Foster  
Principal, Oskaloosa Elementary School  
pfoster at usd341.org



**Past President, Community Relations:** Fred Hollingshead  
Instructional Coach, Shawnee Heights High School  
hollingsheadf at usd450.net, 785-379-5880



**Vice President, College:** Chepina Rumsey  
Assistant Professor, Kansas State University  
785-532-4516



**Vice President High School:** Debbie Sylvester  
Math Teacher, Wamego High School  
sylvesterd at usd320.com, 785-456-2214



**Vice President Middle School:** Liz Peyser, Secondary Math Curriculum Coach, Wichita Public Schools 316-973-4441  
epeyser at usd259.net

**Vice President Elementary:** Josh Cavendar, 6th grade teacher, Joshcavendar at smsd.org



**Bulletin Editor:** Jenny Wilcox, 7th grade teacher, Washburn Rural Middle School, wilcojen at usd437.net

# KATM Executive Board Members

**Zone 1 Coordinator:**

Kathy Desaire, Kindergarten teacher, USD 269  
Kdesaire at usd269.net

**Zone 4 Coordinator:**

**Zone 2 Coordinator:**

Jaclyn Pfizenmaier



**Zone 5 Coordinator:**

Lisa Lajoie-Smith, Instructional Consultant, llajoie at sped618.org

**Zone 3 Coordinator:**

Whitney Czajkowski-Farrell, 7th Grade teacher, Shawnee Heights Middle School,  
Czajkowskifarrellw at usd450.net



**Zone 6 Coordinator:**

Jeanett Moore, 2nd grade teacher, USD 480  
Jeanett.moore at usd480.net

**Co-Webmaster:** Allen Sylvester-  
Science Teacher, Wamego High School

sylvestera at usd320.com, 785-456-2214



**Co-Webmaster:**

David Barnes, Math teacher, Topeka West High School,  
Dbarnes1 at topeka.k12.ks.us