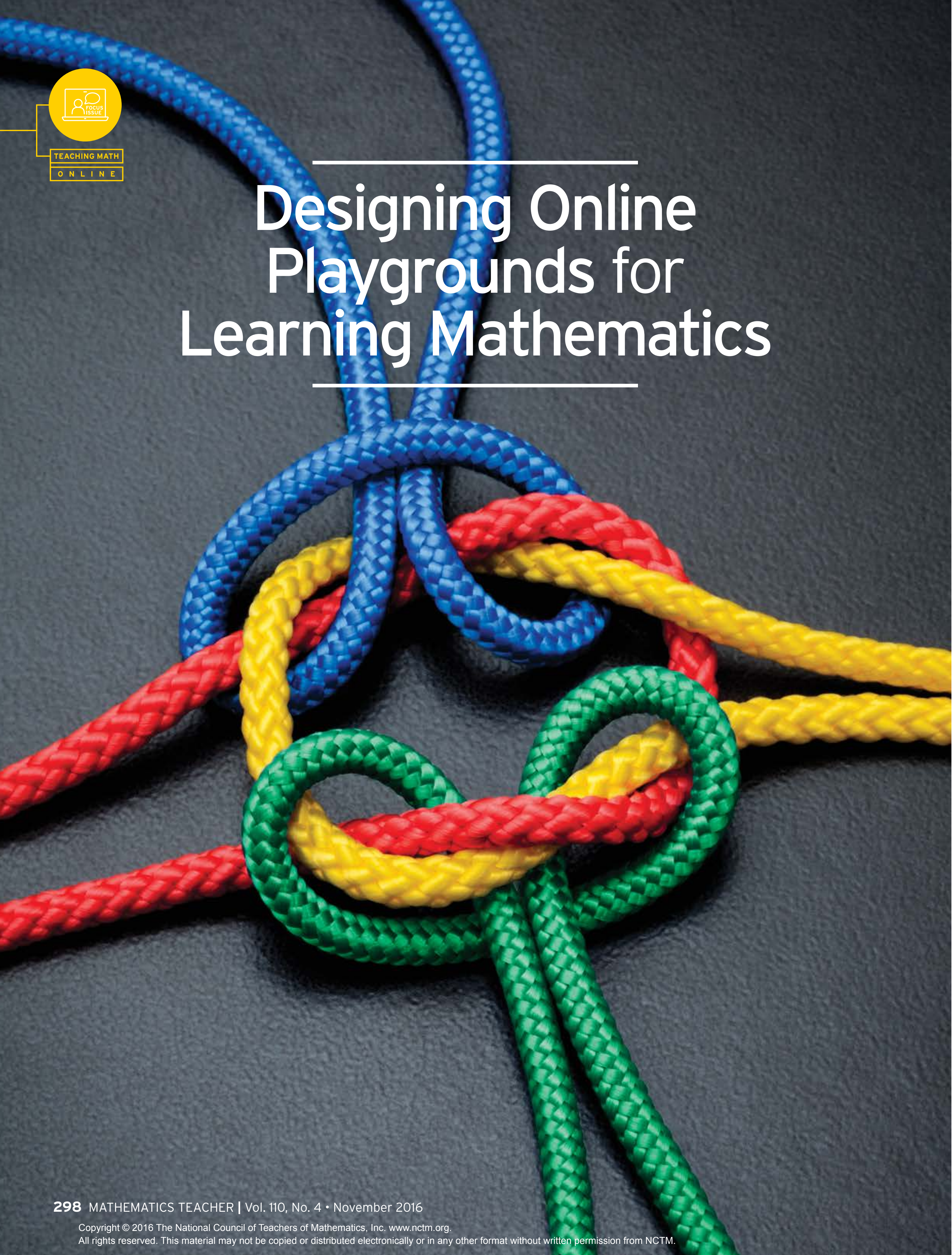

FOCUS ISSUE

TEACHING MATH ONLINE

Designing Online Playgrounds for Learning Mathematics

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Teachers expand conceptions of algebra using interactive tools, video conferencing and social media.

Heather Lynn Johnson, Peter Hornbein, and Dana Bryson

Fully online courses can provide teachers fresh opportunities to expand their mathematical conceptions and infuse technology into their classroom teaching. We share the experience of two classroom teachers (Hornbein and Bryson) who participated in a fully online mathematics education course—Expanding Conceptions of Algebra (ECA)—designed and taught by a mathematics educator (Johnson).

Not only did participating in ECA deepen Hornbein's and Bryson's conceptions of algebra, it also influenced their use of online technology in their classroom teaching. We discuss how participating in an online course, such as ECA, can give teachers a chance to use online technology that they can then incorporate into their own classroom instruction.

DESIGNING ECA: ONLINE PLAYGROUNDS

Johnson designed ECA for practicing teachers to offer them opportunities to deepen their mathematical knowledge and to connect their mathematical knowledge to classroom practice. Rather than attempting to recreate a face-to-face experience in an online course, Johnson designed learning experiences that capitalized on the affordances of the online environment (Dunlap, Verman, and Johnson 2016). Johnson developed *online playgrounds*—instructional experiences with multiple entry points—incorporating three types of playground equipment: online interactive tools, video conferencing, and social media.

Drawing on their ECA experiences, Bryson and Hornbein began incorporating different types of playground equipment into their middle and high school classrooms. We share how the three different types of online playground equipment used in the course can support teachers' and students' learning.

INTERACTIVE TOOLS

We recommend that teachers select online interactive tools that offer multiple entry points for students and provide challenges appropriate for students with a range of prior knowledge. For example, NCTM Illuminations' Pan Balance-Expressions (PB-E) (NCTM 2016), shown in **figure 1**, affords investigation of algebraic expressions with varying levels of complexity. In ECA, Johnson asked teachers to determine connections between symbolic and graphical representations that would be productive for students to make when working with the PB-E activity. Responding, Bryson observed in part that "as the x-value is changed the tracer on each of the graphical representations moves as well as the pan balance." Regardless of the complexity of the algebraic expressions, the connections between the x-values, graphical representations, and pan balance would hold.

Before taking part in ECA, Bryson had found it challenging, and at times frustrating, to find quality online interactive tools to use in her classroom. Participating in ECA fostered Bryson's ability to

JORG GREUEL/GETTY IMAGES

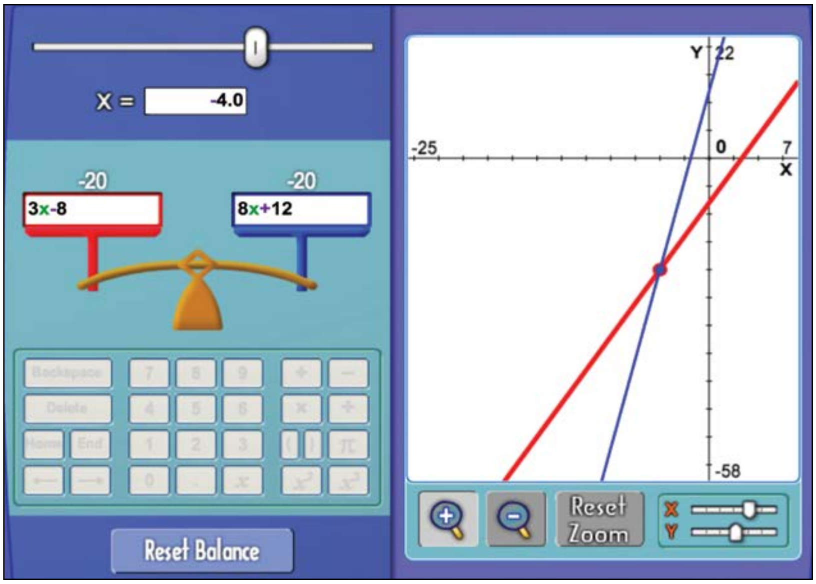


Fig. 1 Pan Balance–Expressions (PB-E), an interactive tool from NCTM's Illuminations website, offers multiple entry points for students.

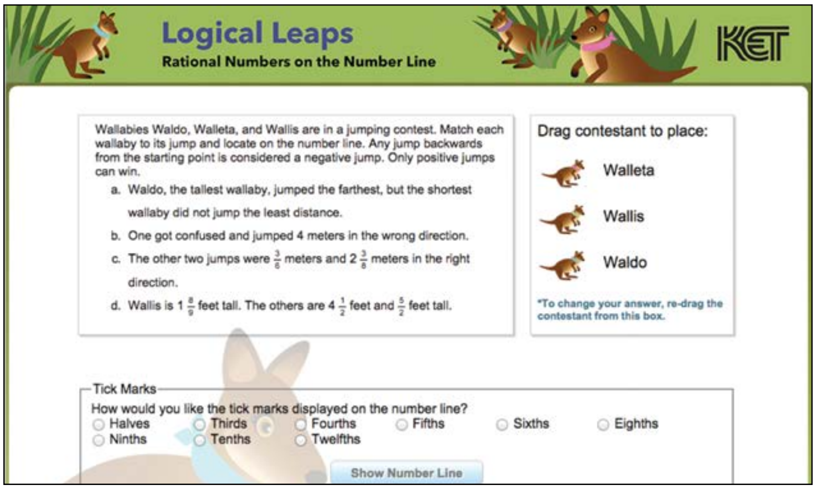


Fig. 2 Co-author Bryson used Logical Leaps (LL) with her middle school students. Logical Leaps © KET (Kentucky Educational Television)

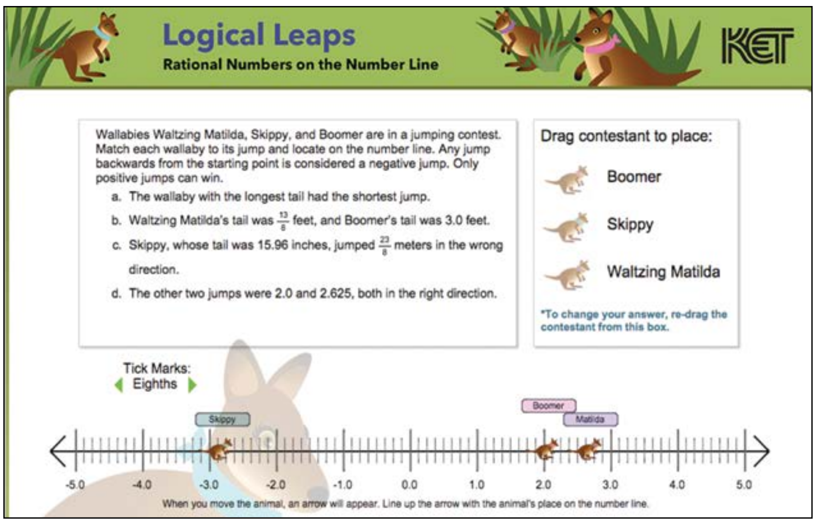


Fig. 3 Students use the given information to place each wallaby on the number line. Logical Leaps © KET (Kentucky Educational Television)

use online interactive tools strategically with middle school students having a range of prior knowledge. One of Bryson's favorite interactive tools—PBS LearningMedia's "Logical Leaps: Rational Numbers on the Number Line" (LL) (PBS LearningMedia 2016a)—is part of the website's Math at the Core: Middle School collection. (Other LL interactive tools focus on different mathematical topics, such as graphing inequalities and adding fractions and decimals.) **Figure 2** provides a screenshot from one of the interactive LL "contests" (PBS LearningMedia 2016b).

The LL activities involve wallaby jumping contests, as well as other contests involving frogs and fleas. In the example shown here, students order wallabies on a number line given information about each wallaby's jump distance, which may be in the form of fractions, mixed numbers, or decimals. Students begin by analyzing given information to determine how to display tick marks and then position wallabies on a number line.

LL offers different levels of complexity. The contest depicted in **figure 2** incorporates distances given in terms of fractions and mixed numbers. The contest depicted in **figure 3** incorporates distances given in terms of fractions, mixed numbers, and decimals, as well as different units of measure for the same attributes (e.g., both inches and feet for tail length).

Interactive tools such as LL can provide opportunities for students to reflect on important mathematical ideas. For example, after working with LL, a student remarked, "I don't understand how a leap can be negative. He's just jumping in the wrong direction." Building from this student's remark, Bryson led the class in a discussion about signed distance and absolute value.

Extending their use of online interactive tools, students can take screenshots of their work, then use those screenshots as sources of reflection. In her middle school classroom, Bryson asks students to take screenshots of their work with LL, then explain how they chose to position the wallabies in a given order. To spark discussion, Bryson shares individual student screenshots with the entire class. For example, given the screenshot shown in **figure 3**, Bryson might affirm that the solution is correct, then ask students how they think a student might have come to position the wallabies in that way. Bryson also presents screenshots of incorrect solutions, without attributing a screenshot to a particular student, to provide students with opportunities to determine which wallabies need repositioning and why.

To gather evidence of students' thinking, Bryson requires students to take multiple screenshots of their work. For example, students take screenshots

JORG GREUEL/GETTY IMAGES

when they check their solution and when they are feeling stuck in their work. When working with LL, students can have difficulty selecting a tick mark length, reflected in screenshots that do not include a number line (see **fig. 2**). Student screenshots provide Bryson with meaningful starting points to use for giving individualized feedback. We view students' screenshots as sources of evidence that teachers can use to add new dimensions to their formative assessment (Schultz and Thunder 2015).

VIDEO CONFERENCING

Communication and collaboration were integral to the ECA experience. Teachers in the course used Zoom video conferencing to converse with Johnson and to work collaboratively with other teachers in the course. Zoom was a great fit for the ECA course because it is easy to access, supports conferences with multiple participants, and allows conferences to be recorded.

Video conferencing can provide opportunities for teachers and students to discuss meaningful mathematics. For example, video conferencing with Bryson helped Hornbein to think deeply about what the slope of a line could represent. To illustrate, we describe one of Bryson and Hornbein's video conferences.

During the video conference, Bryson and Hornbein explored a task requiring them to interpret graphs, such as the one shown in **figure 4**, representing linear relationships between varying parts of yellow and blue pigment in different constant ratios. During the conference, Hornbein came to an important insight: The slope of a line representing a relationship between parts of yellow and blue pigment in constant ratio provides a measure of "greenness." For example, the slope of the line segment shown in **figure 4** is 1/3, indicating a yellower green, because the horizontal and vertical axes represent parts of yellow and blue pigment, respectively.

Using Zoom's recording feature, Bryson and Hornbein created a video record of their conference. Carefully selecting episodes, Bryson and Hornbein then made a shortened video, jointly submitted as an ECA course assignment. Reviewing and editing the video provided opportunities for Hornbein to reflect on his new insight and Bryson to reflect on her use of mathematical terminology.

Video conferencing provides opportunities for teachers and students to interact beyond the bounds of a typical mathematics classroom. Hornbein and his high school students have made extensive use of Zoom's Shared Whiteboard and Screen Sharing features. Recently, Hornbein "Zoomed" with an algebra student, Sian, to support her developing proficiency with completing the square. Using Zoom's Shared Whiteboard, Sian worked to complete the square for different quadratic equations. Viewing Sian's work in real time, Hornbein diagnosed that Sian needed a quick refresher of fraction division. For quadratic equations of the form $ax^2 + bx + c = 0$ with $a > 1$, Sian had difficulty halving improper fractions of the form b/a when a and b were odd and relatively prime. To illustrate, we share Hornbein's work with Sian to complete the square for the quadratic equation $3x^2 + 5x - 12 = 0$. Using Zoom's Screen Sharing, Hornbein demonstrated halving the fraction 5/3 using JavaBars software (Biddlecomb and Olive 2000), available as a free download.

Figure 5 depicts four steps in Hornbein's JavaBars demonstration. Using Zoom's Shared Screen Feature, Hornbein combined two types of playground equipment—a video conference and an online interactive tool—and seamlessly integrated a multimedia review of fraction division into a video

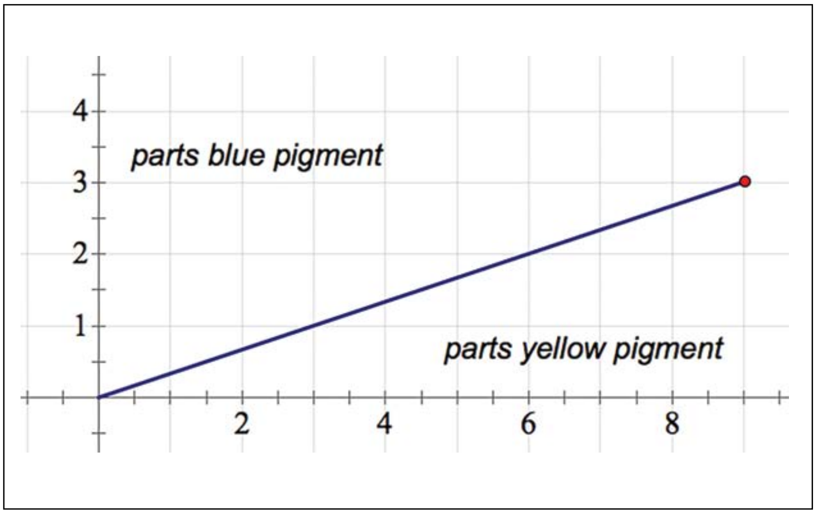


Fig. 4 This graph represents a linear relationship between parts of yellow and blue pigment in constant ratio 3:1.

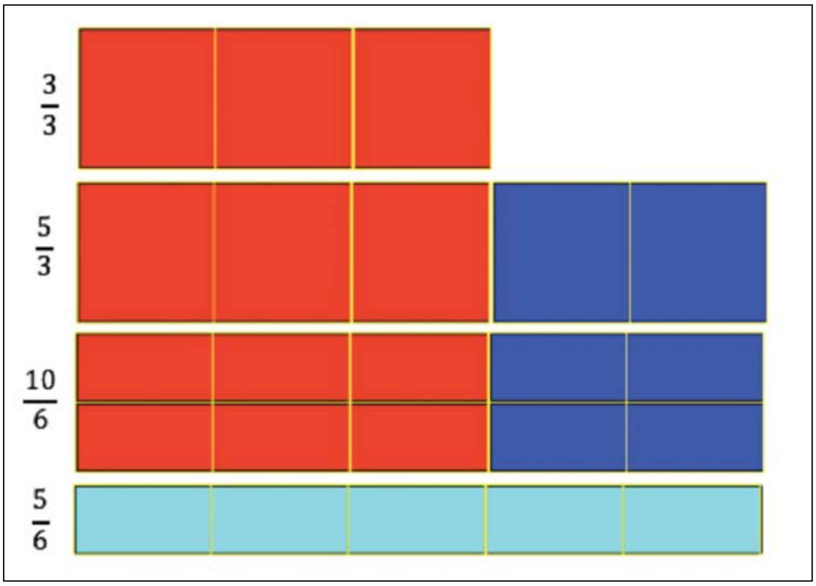


Fig. 5 JavaBars represent 5/6 as half of 5/3.

conference focused on completing the square.

Not only can teachers use video conferencing to interact with students, teachers can provide students with opportunities to use video conferencing to collaborate on homework assignments. For example, Hornbein's students have used Zoom's Shared Whiteboard feature to collaborate to solve homework problems, taking screenshots of their work along the way. We view Hornbein's use of video conferencing as another way that teachers can make mathematics homework meaningful to students (Weiman and Arbaugh 2014).

SOCIAL MEDIA

As part of one ECA assignment, Johnson required that teachers tweet responses to the following question: "How do you know when you need more than one variable?" (See Boaler and Humphreys [2005] for an excellent classroom lesson incorporating this question.) Hornbein tweeted to Johnson: "When you're dealing with more than two quantities."

For teachers not yet using Twitter®, Johnson provided opportunities to respond in other ways, while maintaining Twitter's 140-character limit. Bryson tweeted her response in the ECA course learning management system: "More than one variable is needed when I am being asked to find two distinct quantities."

Because tweets are brief, we recommend providing students opportunities to explain their thinking related to their tweet. For example, Bryson explained that she found it useful for her students to begin by assigning a different variable to represent each distinct quantity, even when it might be possible to represent multiple quantities in terms of just one variable.

Teachers can use Twitter to provide feedback to students. Hornbein discovered a surprising excitement after finally receiving a "favorite" (formerly, a "like") from Johnson in the course. In his high school classroom, Hornbein has found that his students remained engaged for longer periods, trying to receive a sought-after "favorite."

Twitter has provided Hornbein with new ways to interact with students during class and to gather real-time feedback to inform his instruction. Typically, Hornbein provides students with a prompt to which they are to tweet a response. As students respond, Hornbein circulates throughout the room, using the tweets to inform his instruction.

During a geometry class, Hornbein used this prompt: "Explain the differences between a chord, tangent and secant of a circle." One student, Katie, who was typically reserved during whole-class discussions, engaged in a lively Twitter conversation with Hornbein. Initially, Katie tweeted that a chord is a segment that lies in the circle. Hornbein replied, tweeting that Katie should check her definition. Katie responded, tweeting that a chord could not pass through the center of a circle. Subsequently, in Katie's small group, Hornbein clarified that if a chord with endpoints on the circle passes through the center, it would be a diameter. Using Katie's tweets as a source of real-time information, Hornbein adjusted his instruction to support students' learning.

Not only does Twitter afford teacher-student interaction, it also gives students a chance to share mathematical ideas with one another. For example, Hornbein's students have exchanged mathematical tweets when working in small groups. We have found that Twitter can provide students new opportunities to converse and collaborate in a mathematics classroom.

TEACHING TIPS

- **If not all students have access to a particular online technology, be creative.** Given that everyone may not have access to Twitter, we created opportunities for participation without requiring Twitter accounts. By allowing students to respond through email or other means, while keeping to the character limitations of Twitter, teachers can keep the spirit of Twitter alive.
- **Use online technology to broaden opportunities for student participation.** Using online technology in our courses has expanded our conceptions of active student participation. We have observed students who may be more reserved during a class discussion come alive in a Twitter conversation, a video conference, or when working with an interactive tool.

Video conferencing provides opportunities for teachers and students to interact beyond the bounds of a typical mathematics classroom.

Furthermore, online technology can foster students' interaction with their peers as well as their teachers.

- **Use online technology to support formative assessment.** Online technology has provided us with sources of information to make instructional decisions. For example, teachers can draw on students' tweets and screenshots to diagnose students' difficulties, gather evidence of students' thinking, foster classroom discussion, and make real-time classroom adjustments.

PLAYGROUND INTERACTIONS

In ECA, Bryson and Hornbein used online playground equipment—video conferencing, social media, and online interactive tools—to expand their conceptions of algebra. Using the equipment to foster their own mathematical learning helped Bryson and Hornbein infuse online technology into instructional experiences for their middle and high school students.

By inviting students to use online technology as a part a mathematics class, teachers can turn what might have been a classroom management issue into an educational opportunity. Using online playground equipment, students can deepen their mathematical knowledge while engaging in new forms of interactions.

REFERENCES

Biddlecomb, Barry, and John Olive. 2000. "Welcome to John Olive's Home Page." *The University of Georgia Department of Mathematics and Science Education Program in Mathematics Education.* TIMA (Tools for Interactive Mathematical Activity) Development (JavaBars download). http://math.coe.uga.edu/olive/welcome.html

Boaler, Jo, and Cathy Humphreys. 2005. *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning*. Foreword by Deborah Loewenberg Ball. Portsmouth, NH: Heinemann.

Dunlap, Joanna C., Geeta Verma, and Heather Lynn Johnson. 2016. "Presence+Experience: A Framework for the Purposeful Design of Presence in Online Courses." *TechTrends: Linking Research and Practice to Improve Learning* 60 (2): 141–51.

National Council of Teachers of Mathematics (NCTM). 2016. "Pan Balance—Expressions." *Illuminations.* Accessed July 12. http://illuminations.nctm.org/Activity.aspx?id=3529

PBS LearningMedia. 2016a. "Logical Leaps: Rational Numbers on a Number Line." Produced by KET (Kentucky Educational Television). *Math at the Core: Middle School Collection.* PBS & WGBH Educational Foundation. Arlington, VA. http://www.pbslearningmedia.org/resource/mket-math-ns-logicleaps3/logicleaps3/

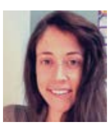
———. 2016b. Wallaby contest. "Logical Leaps: Rational Numbers on a Number Line." Produced by KET (Kentucky Educational Television). PBS & WGBH Educational Foundation. Arlington, VA. http://tdcms.ket.org/cpbmath2/leapsthree/index.html#lineCanvas

Schultz, Kyle T., and Kateri Thunder. 2015. "News & Views: Making Formative Assessment Multidimensional." *Teaching Children Mathematics* 21 (7): 453–54.

Wieman, Rob, and Fran Arbaugh. 2014. "Making Homework More Meaningful." *Mathematics Teaching in the Middle School* 20 (3): 160–65.

Zoom Video Communications. 2016. https://www.zoom.us/





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