ESTABLISHING
FOR MATHEMATI

A grocery shopping problem can link the Common Core's standards with a new classroom culture.

Michelle L. Stephan

Have you ever tried to get a middle school student to explain her reasoning in front of her peers? In your attempts to have students understand one another's reasoning, how many times have you heard, "I don't get it! Any of it!" And how do your middle school students react when someone makes a mistake? Common Core State Standards for Mathematics (CCSSM) expects teachers to establish problem-solving environments in which students create their own solutions to problems, critique the reasoning of their peers, and come to a consensus on viable mathematical strategies and solutions (CCSSI 2010). However, these Standards for Mathematical Practice (SMP) are difficult for most middle school students to enact, and CCSSM gives no information on how to help students embody these practices. This article outlines some teaching strategies from

GEORGE DOYLE/THINKSTOCK (3)

STANDARDS
CAL PRACTICE

my own middle school mathematics classes in an attempt to help my students implement the SMP.

One particular school year, I videotaped my eighth-grade class during the first two weeks of school. The purpose was to document the strategies that enabled students to problem solve and to discuss their solutions without fear of embarrassment. CCSSM was not yet adopted, so research on social norms guided the development of my teaching practice. Social norms are the expectations that the teacher and students have for each other regarding ways of participating in classroom discussions. These social norms, the most prominent in Standards-based classrooms, expect students to—

1. explain their reasoning to others;
2. indicate agreement or disagreement;
3. ask clarifying questions when they do not understand; and
4. attempt to understand the reasoning of others (Cobb et al. 1992; Stephan and Whitenack 2003).

These social norms align with many of the SMP, particularly the standards of making sense of problems and persevering in solving them (SMP 1), reasoning abstractly and quantitatively (SMP 2), and constructing viable arguments and

Fig. 1 Solving the Bacon problem, the first task of the school year, sets the stage for the forms of conversations that students can expect throughout the year.

Drake found a sale on bacon at the Winn-Dixie. They were selling 4 packages for $10.

- How much would Drake pay for 6 packages?
- How much would he pay for 1 package?

critiquing the reasoning of others (SMP 3). The excerpts that follow contain details of some of the strategies I have developed to make these SMPs and social norms come alive in my middle school classroom.

SPECIFIC TECHNIQUES

The first two weeks of the school year were committed to setting social norms through general problem solving (SMP 1). For example, the problem that students encountered when they entered my classroom for the first time is shown in **figure 1**.

At the beginning of the first day of class, the eighth graders were greeted as they walked in. They were then asked their names and asked to sit next to a person who they thought would be helpful during math class. They were also told to try the problem on the whiteboard. Because students had studied ratio and proportions in the seventh grade, I expected this problem to be accessible to all students, including those who were identified with special needs.

As students were problem solving, I *monitored* their work in a fashion similar to that described in *5 Practices for Orchestrating Mathematics Discussions* (Smith and Stein 2011). During my monitoring time, students were encouraged to write their thinking on paper and to talk with a peer when they were stuck (SMP 1). I also gathered information on how students solved the problem so that I could *select* students who had solution processes that would be helpful for the follow-up class discussion.

Because of my agenda of establishing social norms, students presented their ideas in a specific sequence. For example, Brianna went first because her solution method was highly computational and would need deeper explanation for other students to understand. Her presentation would give the class the opportunity to discuss the importance of explaining and asking questions. The six strategies that follow guide the establishment of social norms for productive problem solving.

Strategy 1: State Expectations Early

That opening period would be the first whole-class discussion of the year, so it was critical to use it to begin the process of developing the norms that would drive the students for the rest of the year. Therefore, I engaged them in a discussion about ways to participate in the upcoming discussion. They were told that their job is to listen. When students were prompted to explain why, Anders responded, "You might like their ideas." To this statement, I added the following comments:

> You might like their ideas. And then you should steal it, right? Steal their ideas? That's why I want you to listen. I'm going to be asking you when somebody's up here presenting, Arthur, I want you to be listening, trying to understand their way. What if you didn't do it their way? Should you try to understand their way? Or just say, "Who cares? They didn't do it *my* way, I don't need to listen." No! You need to listen because as Anders just said, you might like their way better, right?

I also added this comment:

> I'm going to be asking you questions. Let's say that Brianna is up here explaining. I'm going to ask someone to explain what Brianna just did. And if you can't, you need to ask a question.

The expectation that students should listen to others' explanations was explicitly outlined. I asked them specifically what their "job" is when someone is explaining, then solicited reasons why this was important. However, if the SMP of analyzing and critiquing other students' arguments is going to become routine, stating these expectations upfront is *not* enough. Teachers must hold students accountable for listening, so I stated my accountability technique: *I am going to call on students* to explain what another student just argued.

Strategy 2: Hold Students Accountable for Explanations

To begin the whole-class discussion, Brianna came to the board to explain her approach to solving the Bacon problem in **figure 1**. At this prompt, Brianna realized I was expecting more than just an oral presentation from her desk. Our class conversation can be found in **figure 2**. I again called on the class to anticipate the rationale for certain expectations. Arthur argued that it is easier for the speaker just to read it, so I asked about the needs of the "audience." Anders explained that it would be easier for the audience to understand if Brianna wrote something, and she complied (see **fig. 3**).

As Brianna wrote her process on the board, I challenged the students to see if they could figure out her reasoning *while* she wrote it. This was my way of communicating to students that they should be analyzing Brianna's work and attempting to understand it (SMP 3). I paused for a few seconds of thinking silence and asked Brianna to move to the side so that *everyone* could analyze her written argument (SMP 3). The statements, "I think *we're* ready" and "Can you explain . . . to *everybody,*" rather than "I think *I* am ready" or "Can you explain . . . to *me*?" are ways to let students know that the purpose of explaining is for the *students* to analyze and verify solutions, not just the teacher.

As a result of this prompting, Brianna said:

> I knew that 4 packages was $10.00. So I did 4 divided by $10.00 and got $2.50, so one package was $2.50. And then I knew I had to get 6 packages so I added the four and one was $2.50 plus $2.50 was $5.00. And $10.00 plus $5.00 is $15.00.

Strategy 3: Hold Students Accountable for Asking Questions

The computational explanation given by Brianna did not contain an indication of what her steps and the results meant, so I searched the room to see what sense students had made of her work. I asked students to indicate whether they agreed or not by asking:

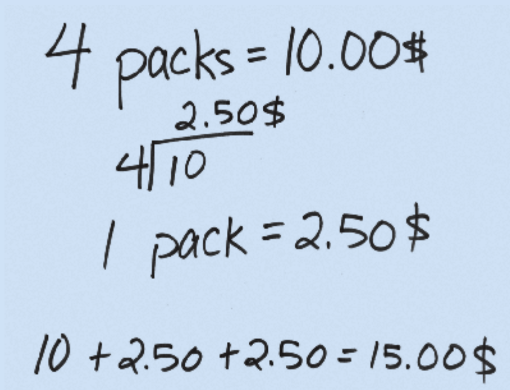
> Could you re-explain it? [They indicated yes.] Hold your hand up if you've got a question. Before she escapes [sits down], anybody got a question for her? That means if I call around, you'll know, you'll be able to explain her way, right? Alright. I'm going to call around. Valerie, you're her partner. What'd she do, how'd she solve this one?

Valerie responded, "I don't know." Because Brianna's explanation was very procedural (see Thompson et al. 1994), many students probably did not connect why she added $2.50 twice to $10.00. This procedural explanation gave me the opportunity to hold students accountable for asking questions when they did not understand. I let students know that I was about to ask others to re-explain Brianna's solution method. As expected, no one raised a hand, so I called on Valerie to re-explain. When she admitted she could not, I took this time to re-iterate my expectation that students should raise their hand if they do not understand. "Does anyone have a question?" is a weak way to hold students accountable because they can answer "yes" and the teacher moves on. The teacher should follow up and ask specific people to re-explain; if they cannot, reiterate the expectation.

Fig. 2 This conversation demonstrates how the explain-your-thinking expectation became the norm.

Brianna: Oh, I have to write it, too?
Teacher: Well, what do you guys think? Should she write something on the board, or should she just read it from her paper?
Arthur: It's easier to read it from your paper than write it.
Teacher: It *is* easier when you're the speaker. Right, Hugo? To just speak it from your paper? What about if you're the person listening? What's easier for you if you're trying to learn?
Anders: It's easier [for the listener] to read it.
Teacher: It's easier to read it, Anders says. So what do you think? What's your advice to her? Should she write something down for you to read?
Students: Yeah!

Fig. 3 At the request of classmates, Brianna demonstrated her reasoning by writing the explanation on the board.



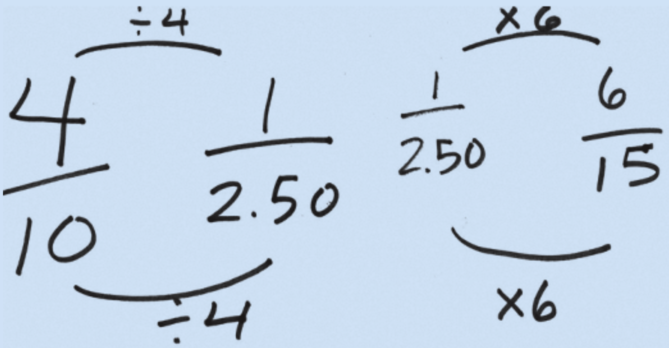
Strategy 4: Hold Students Accountable for Making Sense of Solutions

Asking questions is important for helping students understand others' solutions, but the teacher must help students know what questions to ask. Most students give very calculational explanations (i.e., the steps that were taken) for their solutions (Thompson et al. 1994). The teacher's role is to push students toward more conceptual explanations in which the student explains *why* a particular calculation was made and what the results of that calculation mean in terms of the quantities in the problem situation (SMP 2). Thompson and others (1994) contend that conceptual explanations are more beneficial for struggling students

Fig. 4 In this discussion, the teacher pushed the students to expound on the calculations.

Jamie: OK, so when I first looked at the problem, it said that 4 packages was $10. So 4 and 10. And then I tried to get to 1. I divided 4 by 4 to get 1 and 10 by 4 to 2.50. Then I did 1 times 6 to get 6 and then 2.50 times 6 to get to 15 [see **fig. 4b**].
Teacher: Alright; Judson, what do these numbers on the top stand for in the problem, for Jamie? What are they standing for?
Judson: The number of packages.
Teacher: How many packages? Mariana, what were you gonna say?
Mariana: Wouldn't it be like finding the unit rate?
Teacher: Woo! Did you hear that? What do you think? That was kind of directed at you [class]? What did she just ask? What did Mariana just ask? [No one raises their hand.] Oh no! All listen. Mariana, ask again real loud.
Mariana: Wouldn't that be finding the unit rate?
Marta: That one package costs 2.50.
Teacher: That one package costs 2.50. There it is. And then you used that unit rate, Jamie, to find out how much 6 packages cost. What do you think about Jamie's way?

(a) Class discussion



(b) The calculations in Jamie's strategy

Fig. 5 In this excerpt, the teacher helped students push through and reach understanding.

Teacher: Oh, what part don't you get?
Keisha: The whole thing.
Teacher: Oh no! The whole thing! Let's start here. Do you understand that part?
Keisha: Uh, 4 over 10? That four, that four packages cost $10.00.
Teacher: You understand it, that part! [with excitement] What's this part all about, Keisha? See if she can get it. I bet she can, don't you? What's that next part all about?
Keisha: Does it say 250?
Anders: That's 2 point 50.
Keisha: Oh! That one pack costs $2.50.
Teacher: And then what? What's this stand for?
Keisha: That's one package, and it costs $2.50.
Teacher: Uh-huh. What's this stand for?
Keisha: 6 packages and $15.00?
Teacher: Uh-huh.

because the reasons behind the steps are revealed to them.

To begin the sample discussion in **figure 4a**, Jamie simply restated the steps that he used to solve the problem. To press for understanding, I asked Judson what the numbers on the "top" stood for, so that students could draw connections to the quantities in the Bacon problem. As a result, Mariana brought out the term *unit rate*, and Marta related to students what that meant in this situation.

Strategy 5: Hold Students Accountable to Question What They Do Not Understand

Often when students do not understand the explanation of another student, the teacher presses them to ask a question. At the beginning of the school year, the most frequent response in my class is "I don't get it! Any of it." How does a teacher handle this exclamation without re-explaining everything in his or her own words? How can we teach students how to identify their areas of misunderstanding?

As the dialogue in **figure 4a** continued, Keisha responded that she really did not understand Jamie's solution. In the subsequent exchange with Keisha, shown in **figure 5**, I attempted to model how a student can go through the steps of the process and how to ask themselves, "Do I understand this part?" As it turned out, Keisha was able to explain almost all the steps except when misreading Jamie's writing as 250 rather than 2.50. Such a misinterpretation caused her difficulty in making sense of the rest of the solution. Helping to find the misunderstanding was an effective strategy when students indicated that they did not understand an entire solution. With enough of these explicit conversations, students began to analyze others' solutions to find specific places about which they could ask questions rather than say they misunderstood the entire solution.

Strategy 6: Praise Students for Their Participation and for Providing Informative Feedback

To end the first discussion of the school year, it is extremely important to stop in a similar manner as started: being explicit about expectations for current and future participation. This helps students understand the characteristics of their participation that were acceptable in class and those that needed to be improved. In addition, it is important to point out which social norms and Standards for Mathematical Practice were not acted on very well. For example, at the close of the Bacon problem, students were praised for their willingness to present their arguments in front of their peers, acknowledging how nervous it might have made them. The class applauded each person who presented and congratulated students on listening and being able to re-explain students' arguments. I also stated that we had some work to do asking questions when students do not understand one another. However, we would begin working on that more during the next class period.

ESTABLISHING NORMS TAKES TIME

The objective of this article was to describe the strategies (see **fig. 6**) for establishing social norms that are consistent with CCSSM's Standards for Mathematical Practice. A classroom environment in which students persevere in solving problems and feel engaged and safe enough to explain their thinking to their peers can have a positive effect on their learning (Tarr et al. 2008). However, social norms for communicating productively in class do not arise fully formed on the first day of class. It takes weeks for the teacher and students to establish these norms and for them to become stable for the rest of the school year. For this reason, I do not attempt to establish every single norm on the first day.

I reserve the first two weeks for building strong social norms and SMPs. Two weeks is not only a reasonable time period but also essential for setting the stage for communication in later units that target specific content. I recommend using general mathematics problems that elicit a variety of strategies and are not focused on developing new knowledge in a particular domain. Doing so allows the teacher to focus his or her attention explicitly on establishing

Fig. 6 This listing describes strategies for establishing standards-based social norms in math class.

Strategy 1: State expectations before the first explanation occurs
The teacher—
- states his or her expectations explicitly before a whole-class discussion begins; and
- engages students in developing the rationale for some of the norms.

Strategy 2: Hold students accountable for explaining
The teacher—
- calls on as many students as possible and uses names on day one;
- calls on students by name to explain their reasoning; and
- expects students to explain both verbally and in written form while engaging students in understanding the rationale.

Strategy 3: Hold students accountable for asking questions
The teacher—
- calls on students *by name* to see if they have questions;
- asks students to re-explain a solution method; and
- reiterates that it is unacceptable to violate certain norms (e.g., not asking a question when they do not understand).

Strategy 4: Hold students accountable for making sense of solutions
The teacher—
- asks students to not only re-explain but also describe why the particular procedures were taken and what the results mean in terms of the situation.

Strategy 5: Hold students accountable to question what they do not understand
The teacher—
- does not accept students' claims that they do not understand the entire solution explanation and instead teaches them how to analyze the strategy to find the misunderstanding.

Strategy 6: Praise students for their participation and for providing informative feedback
The teacher—
- applauds students for meeting norm expectations and provides feedback when certain norms are not being met effectively.

problem-solving norms rather than on developing students' knowledge of one particular mathematical concept.

CCSSM Practices in Action

SMP 1: Make sense of problems and persevere in solving them.
SMP 2: Reason abstractly and quantitatively.
SMP 3: Construct viable arguments and critique the reasoning of others.

REFERENCES

Cobb, Paul, Terry Wood, Erna Yackel, and Betsy McNeal. 1992. "Characteristics of Classroom Mathematics Traditions: An International Analysis." *American Educational Research Journal* 29: 573–604.

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Smith, Margaret S., and Mary Kay Stein. 2011. *5 Practices for Orchestrating Mathematics Discussions*. Reston, VA: National Council of Teachers of Mathematics.

Stephan, Michelle, and Joy Whitenack. 2003. "Establishing Classroom Social and Sociomathematical Norms for Problem Solving." In *Teaching Mathematics through Problem Solving: Prekindergarten–Grade 6,* edited by Frank K. Lester, pp. 149–62. Reston, VA: National Council of Teachers of Mathematics.

Tarr, James E., Robert E. Reys, Barbara J. Reys, Óscar Chávez, Jeffrey Shih, and Steven J. Osterlind. 2008. "The Impact of Middle-Grades Mathematics Curricula and the Classroom Learning Environment on Student Achievement." *Journal for Research in Mathematics Education* 39 (May): 247–80.

Thompson, Alba G., Randolph A. Philipp, Patrick W. Thompson, and Barbara A. Boyd. 1994. "Calculational and Conceptual Orientations in Teaching Mathematics." In *Professional Development for Teachers of Mathematics,* 1994 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Douglas B. Aichele and Arthur F. Coxford, pp. 79–92. Reston, VA: NCTM.

Any thoughts on this article? Send an e-mail to ***mtms@nctm.org.****—Ed*.

Michelle L. Stephan, michelle.stephan@uncc.edu, is a former middle school mathematics teacher who now works as an assistant professor of mathematics education at the University of North Carolina at Charlotte. She works with elementary, middle, and high school mathematics preservice teachers and is interested in designing inquiry mathematics materials for all students.