KATM Bulletin

Kansas Association of Teachers of Mathematics

October 2016

2016 KATM Annual Mathematics Conference

#thestruggleisreal

October 14, 2016

8:00 AM to 3:30 PM

Maize High School

11600 W. 45th St. N.

Maize, Kansas 67101

Join us at our annual conference for Kansas mathematics educators to participate in engaging and interactive sessions. This year we will be using a new format.

PreK-5th grade:
AM- with keynote, Chris Shore
PM- choice of breakout sessions

6th grade-HS:
AM- choice of breakout sessions
PM- with keynote, Chris Shore

Visit katm.org for details on registration, online payment and T-shirts.

- $50 per participant, lunch included
- Half day AM or PM option, $20 per participant, lunch NOT included

Inside this issue:

A Message from our President

Hello Kansas Math Teachers!

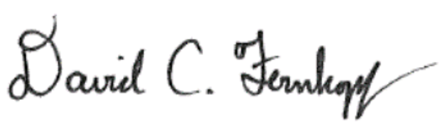
My name is David Fernkopf and I am the new KATM president. I am currently a principal at Overbrook Attendance Center, part of the Santa Fe Trail School District. This is my fifth year as a principal, and it is my seventh year being involved with KATM. I have served as a zone representative, treasurer and now your president. I am excited and happy to take on this role.

In this issue of the KATM Bulletin we offer information on Numbers and Operations in Base 10, Ratios/Proportional Relationships, and Number and Quantity.

We have some very talent people on our KATM board who have offered articles for this newletter.

We also are very fortunate in having a partnership with NCTM. This partnership with NCTM lets utilizes some of their national publish articles in our own newsletter. You will also see these article throughout the newsletter.

So take some time and see what information KATM has to share with you.



David C. Fernkopf

President, KATM

davidfernkopf@katm.org

P.S.

If you want to become more involved with KATM? We do have a few zone positions open. Send me an email and I can tell you how to become more involved!

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Dear Kansas Math Teachers,

Greetings from a new school year! I hope that these first several weeks of school have been great. The KATM board has been working hard to prepare a Bulletin that will benefit all Kansas teachers. Last spring, we finished our series of Bulletins about the Mathematical Practices. With this Bulletin, we launch a new series of Bulletins that examine CCSS standards from a content standpoint. We will use the progressions to examine how the standards grow and progress over grade levels.

In trying to make our Bulletin more useful for people, we have started putting hyperlinks with the Bulletin. You can link to the article that you want directly from the table of contents, or click a link within our document to a website. We hope this change will make it easier for our members to quickly locate what they are looking for in our Bulletin.

As always, we would love to hear from our members about other things you would like to see in future Bulletins to make it more useful. You can email me at jennywilcox@katm.org with any suggestions. Or, we would love to feature articles and lesson plans from our members if you have an awesome idea you'd like to share!

Sincerely,

Jenny Wilcox

KATM Bulletin Editor

Join us for our annual KATM meeting session at our upcoming KATM fall conference. Hear the latest KATM news and happenings before you head out for some awesome breakout sessions!

SAVE THE DATE!

The KATM conference is returning to Topeka in 2017 at Seamen Middle School on October 16, 2017.

Join us!

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Common Core State Standards Mathematics Standards Progressions

Kindergarten	1	2	3	4	5	6	7	8	High School
Counting and Cardinality									Number and Quantity
Number and Operations in Base 10						Ratio and Proportional Relation-			
			Number and Operations:			The Number System			
Operations and Algebraic Thinking						Expressions and Equations			Algebra
								Functions	Functions
Geometry						Geometry			Geome-
Measurement and Data						Statistics and Probability			Statistic and Probability

Last year, KATM wrapped up our series on the mathematical practices. This year, we begin a new series, focused on the standards progressions. We will be focusing on how topics progress and change over the K-12 curriculum.

October 2016: Number and Operations in Base Ten to Ratios and Proportional Relationships to Number and Quantity

December 2016: Number and Operations: Fractions to The Number System to Number and Quantity

February 2017: Operations and Algebraic Thinking to Expressions and Equations/Functions to Algebra and Functions

April 2017: Geometry

October 2017: Measurement and Data to Statistics and Probability

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Number and Operations in Base Ten to Ratios and Proportional Relationships to Number and Quantity

Number and Operation in Base Ten (K-5)

Kindergarten: Work with numbers 11-19 to gain foundations for place value.

1st grade: Extend the counting sequence.

Understand place value.

Use place value understanding and properties of operations to add and subtract.

2nd grade: Understand place value.

Use place value understanding and properties of operations to add and subtract.

3rd grade: Use place value understanding and properties of operations to perform multi-digit arithmetic.[1]

4th grade: Generalize place value understanding for multi-digit whole numbers. Use place value understanding and properties of operations to perform multi-digit arithmetic.

5th grade: Understand the place value system. Perform operations with multi-digit whole numbers and with decimals to hundredths.

Ratios and Proportional Relationships (6-8)

6th grade: Understand ratio concepts and use ratio reasoning to solve problems.

7th grade: Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System (High School)

The Real Number System: Extend the properties of exponents to rational exponents.

Use properties of rational and irrational numbers. **Quantities: Reason quantitatively and use units to solve problems.**

The Complex Number System: Perform arithmetic operations with complex numbers. Represent complex numbers and their operations on the complex plane. Use complex numbers in polynomial identities and equations.

Vector and Matrix Quantities

Represent and model with vector quantities. Perform operations on vectors. Perform operations on matrices and use matrices in applications.

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Opportunities to Develop Place Value through Student Dialogue

By Amy R. Kari and Catherine B. Anderson

The problem 11 + 9 was written on the board at the front of the room. Eleven first graders and nine second graders sat on the carpet, their facial expressions intent as they thought about solutions. I had asked them to try to think of strategies they could use that did not involve counting on their fingers. They did not use paper and pencil because this was what we call "Mental Math" time at our school.

As the students finished their mental calculations, they began to grin and raise their hands. Some began to whisper. I quietly reminded them, "Please wait for the people still working. Be patient. Put your thumb up in your lap to show you are finished, so your arm won't get tired and fall off." The students giggled, but they quieted down until everyone had their thumbs up. Then I asked for answers.

Sharing Computational Strategies

I recorded the two answers the students gave me on the board. Some students thought the answer was 20, others 29. For each answer, a spirited chorus of "Agree" or "Disagree" sounded. I called on Breanna to explain her strategy for solving the problem. She confidently told the class that she started at eleven and counted on nine more to get twenty. I wrote Breanna's name and "Counted on nine more from eleven to twenty" on the board. I always label the counting strategies that students use, such as counting on, counting all, or skip counting, to familiarize students with the vocabulary that describes their actions.

Next, I asked if anyone who got the answer 29 was willing to share his or her thinking. A couple of students raised their hands. I called on Alejandro. He said, "I did ten plus ten equals twenty, plus nine more equals twenty-nine."

Immediately, there was a chorus of disagrees and agrees. Someone called out, "Where did you get your tens?" My students are very active during mathematics. I encourage them to share their ideas, their agreement or disagreement with others' ideas, and their questions. The dialogue that takes place each day during mathematics drives their learning. Over time, they become confident that their ideas have value and that they can learn from their mistakes.

In response to the question about where he got the tens, Alejandro said, "From the ones in the eleven."

I pointed to the one in the tens place and clarified: "So this one is a ten?" He nodded. Pointing to the one in the ones place, I asked, "And this one is a ten?" He nodded again. I turned to the class. "Do you agree with Alejandro?" The students started to nod their heads. "Are you sure?" I asked. Some students said yes and others said no; I saw a lot of puzzled faces.

I pointed again to the one in the tens place and asked, "Is this a one or a ten?" Most of the students said it was a ten. A few of the first graders and one of the struggling second graders, who did not understand place value, said that it was not a ten.

Haley called out, "They are both tens. I agree with Alejandro."

"There are no tens," said Emily, a shy first grader. "They are both ones. There are no zeros."

Aleigha said, "We can add a zero to any one and make a ten."

"You can't just add a zero when you want to," Taylor argued. "It has to be there."

"There has to be a ten 'cause if you count backwards from eleven, you say ten," Marcos said. "So it has ten in it."

Lien said, "Yes, if you take one away from eleven it's ten."

Joshua said, "They can't both be tens. Then it would be twenty. See: Ten plus ten is twenty. That's too big. The first one is a ten. The other one is a regular one. Ten plus one is eleven."

Haley came back into the conversation. "Now I agree with Joshua and Lien. It can't be two tens. I think eleven is ten plus one too."

Most of the students started to nod their heads to show that they agreed with Haley. I asked Lien to share her strategy. She told the class, "I took the regular one from the eleven and put it with the nine and that's ten. Then I did ten plus ten is twenty. The answer is twenty."

I turned to Maria, a second grader with limited English skills. I pointed to the equations I had recorded while Lien was talking and asked, "Is one plus nine ten?" She nodded. I asked, "Is ten plus ten twenty?" She nodded again. "So you agree?"

Maria smiled and softly said, "Agree." When I asked if Maria had done the same thing as Lien did, she smiled again. Maria was developing place value and had good strategies for solving problems. However, she did not know the English words to explain her thinking to the class. I often told her to write her strategy on the board for her sharing and then asked the other students to try to explain what they thought she did. I also called on her often to give yes or no responses so that she felt she was part of the conversation.

Because the bell rang for recess, we stopped our conversation about the number eleven for the day. I knew we would return to it again because I had discovered that the number eleven can be very confusing to students who are developing their understanding of place value.

The second graders in the group had struggled mathematically as first graders. Most of them did not conserve number when they left kindergarten. They did not become fluent with sums to twenty as first graders and had no understanding of place value at the beginning of second grade.

The Importance of Repeated Discourse

A couple of weeks after our initial conversation about eleven, I asked my students to solve 11 + 11 + 11. The strategies that I recorded on the board demonstrated that many of them were progressing in their thinking. The students gave four answers: 33, 60, 32, and 31. The answers of 32 and 31 came from miscounts by first graders like Emily. Alejandro stuck with his idea that all the ones were tens, as did a couple of other students, and got

60. Both Haley and Aleigha, however, showed significant progress by using the strategy 10 + 10 + 10 = 30, 1 + 1 + 1 = 3, and 30 + 3 = 33. They both were able to articulate where they got their numbers. Joshua asked me to rewrite the problem as 10 + 1 + 10 + 1 + 10 + 1 = 33 before using the same strategy as Haley and Aleigha. After sharing their strategies, the students engaged in another spirited debate about the meaning of the digits in eleven.

The numbers twenty-one, thirty-one, and so on pose the same problems for students who overgeneralize the idea that if there is a one in a two-digit number, it must be a ten. One year, I had a student named Mai who told the class that the one in thirty-one is a ten. Timmy told her, "No way! It's just a one and a thirty." Seven students agreed with Timmy, but twelve agreed with Mai. Sara said it had to be thirty and one because she knew that thirty plus one equals thirty-one and ten plus three is only thirteen. Some students said a student teacher had told them that when one digit in a two-digit number is a one, the one is always a ten. I have no idea what the student teacher actually told them. A lot of dialogue and many experiences are necessary to clear up such misconceptions.

I wrote the number twenty-one on the board and asked Mai to count out twenty-one cubes. I asked her, "Could you make some piles of ten and match your cubes to the numbers on the board?"

Mai carefully counted out twenty-one cubes. She made two stacks of ten and had a single loose cube. Initially, she took one of the groups of ten and put it by the numeral one. She put the other group by the two and said, "This one is left over," referring to the single remaining cube. I did not say anything. Then Mai moved both groups of ten by the one, but that did not satisfy her either. Suddenly, she looked at me and said, "I know." She moved both groups of ten by the two and the single cube by the one. She smiled and said, "I get it."

Indeed, Mai did understand, and she continued to understand from then on. Other students may not solidify their understanding so quickly. Developing place value can take a long time. Students who seem able to articulate clearly their understanding of our number system one moment may seem confused the next. At our school, we say these students are "transitional" in their understanding of place value. We make sure that we hold many conversations during mathematics time that will help these students solidify their understanding. Even though we may do many other mathematics activities, conversations about tens recur throughout the year. We do not simply do one addition unit or place-value unit and move on.

Laying a Foundation for Conversations about Place Value

Long before we discussed the mysteries of the number eleven, my students were working on building their understanding of place value. We started our work on place value at the beginning of the year. The students spent a lot of time playing games that helped them become fluent with all the sums, first to ten and later to twenty. Playing board games was one popular activity. (A file folder with a path marked by colorful stickers makes an easily stored game board; Unifix cubes are game markers.) The children rolled a die and then moved the number of spaces that represented the difference between the number on the die and either ten or twenty. For example, if students roll a three, they move seven spaces. Another way to play is to use two or more dice and roll them, find the sum, and move that number of spaces. Teachers can make the game more complicated by using number cubes instead of dot dice (children may become too reliant on counting the dots on the dice) or polyhedra dice with higher numbers.

During our Mental Math time, we spent a lot of time working with multiple addends that contained sums of ten. One such problem, 5 + 6 + 4 + 5, produced a variety of strategies. Guillermo recognized the 5 + 5 as ten, then counted on six more, then four more to get twenty. Emily counted all four numbers. Several students used the strategy of 5 + 5 = 10, 6 + 4 = 10, 10 + 10 = 20. Aleigha shared her strategy of 5 + 6 = 11, 5 + 4 = 9; she counted to find both sums. Then she was faced with 11 + 9 and counted on from eleven. When I asked her if this had been her original strategy, Aleigha admitted that she had originally counted as Emily had, but she liked this method better. Students often create new strategies during sharing time so they have a chance to share.

Rafael confused the class with his strategy. He started with 6 + 6 = 12. Students asked, "Where did the other six come from?" and noted, "There aren't two sixes." Rafael explained that he got his second six by adding the five and a one taken from the four. He finished by adding the second five and the remaining three from the four to get eight. Then he put eight and twelve together by counting on to twenty. Even though the strategies were not very efficient, I was pleased that the students were beginning to play with numbers more often and use combinations that they knew well.

When I gave the students a problem such as 12 + 4, many of them were comfortable adding two to twelve to get fourteen and then adding the remaining two. Just to put a new idea in their heads, I offered an alternative: "One of my students once said that you could put the two from the twelve with the four, like this [I wrote 2 + 4 = 6], and get six. Then they told me to add the ten from the twelve to the six and you get sixteen. Do you think this person was right or wrong?" I got very little response from the students and left the discussion unfinished.

A few days later, however, Dylan used a ten to solve 12 + 6 + 2. He told the class, "2 + 2 = 4, 6 + 4 = 10, ten plus the ten from the twelve is twenty."

I asked the class if there were tens in twenty. Mark said, "Yes, the two is like ten plus ten is twenty; that's two tens." After my alternative suggestion, the students' talk about tens came naturally. I always made sure to question the meaning of what the students were saying and to ask more than one student to put things in their own words. "Kid language" seemed to make more sense to the students than did adult explanations.

As we progressed in our work, I gradually increased the numbers. As the numbers moved into the double digits, some students continued to use their counting strategies, but others began to talk more confidently about place value. For the problem 12 + 12, Emily actually experimented by using 2 + 2 = 4, 1 + 1 = 2, and 4 + 2 = 6. Most of the students disagreed with her, so I asked her to pay close attention to their comments. Aleigha said, "No, those ones are really tens. So it should be 10 + 10 = 20, 20 + 4 = 24."

Chris commented, "Yeah, tens because you've passed it. If you count to twelve, you have to pass ten." Meanwhile, Emily tried to count on her fingers and reluctantly agreed that 24 was the correct answer.

Chris became a leader in the tens talk. His explanations and those of other strong students influenced the development of the group. Other students started to think about tens and agreed that there were tens in the greater numbers. These children did not use tens to solve problems, however. Each student eventually reached his or her own breakthrough point.

One day, T'ni described her strategy for the problem 10 + 13 + 7: "I got this from Chris. I did 10 + 10 = 20, 7 + 3 = 10, 10 + 20 = 30."

At about this time in the group's development, I introduced the problems 11 + 9 and 11 + 11 + 11. Once most of the students had resolved their conflicts about the values of the digits in the number eleven, they started to make great progress.

Using Multiple Problems to Differentiate Instruction

Some of the students were having a more difficult time with place value than the others were. I began to use two problems, one easier than the other, during Mental Math, and I let the students choose to do one or both problems. For example, one day I put up the problems 26 + 13 + 4 and 44 + 26 + 36. I called on Emily first, because she had been developing a tendency to simply repeat someone else's strategy as her own. I wanted to encourage her to think for herself. I let her know in advance that I would call on her first. She was ready. I recorded her strategy for 26 + 13 + 4 as she shared it: 20 + 4 = 24, 24 + 6 = 30, 30 + 10 = 40, 40 + 3 = 43. I was thrilled that she had come a long way in her understanding of place value and her ability to effectively solve problems.

T'ni, another student who had been struggling, chose the same problem and showed how much she was progressing when she shared her strategy: 20 + 10 = 30, 30 + 6 = 36, 36 + 4 = 40, 40 + 3 = 43. Both girls were confident in their explanations and proud that their classmates agreed with their thinking. I was impressed by the variety of strategies that the group was producing.

Students had a range of successful strategies for the second problem of 44 + 26 + 36 as well. Aleigha told the class, "I know 40 + 20 + 30 = 90. 90 + 6 = 96, 96 + 6 = 102, and 102 + 4 = 106."

I probed for a more detailed explanation of a couple of Aleigha's steps. "How did you know that 40 + 20 + 30 = 90?"

She replied, "I know 40 + 20 is 60, plus 30 more is 90."

Then I asked, "What did you do when you got here?" I pointed to the step 96 + 6 = 102.

She said, "I counted on from 96 to 102."

I asked the students if they could think of a way to add 96 and 6 without counting on. After a moment of reflection, Joshua raised his hand. "Take the six from the ninety-six and put it with the other six. That's twelve. Take the ten from the twelve and put it with the ninety. That's one hundred. Now put the two from the twelve with the one hundred. That's one hundred two."

Conclusion

Principles and Standards for School Mathematics (NCTM 2000) recommends that all K–12 students should be able to "organize and consolidate their mathematical thinking through communication" and "analyze and evaluate the mathematical thinking and strategies of others" (p. 60). Our Mental Math sessions are very stimulating because the students focus their explanations on how they can decompose numbers to facilitate finding solutions. They explore with numbers and become very flexible in their thinking. They build strong mathematical vocabulary to describe their thinking.

Too often, teachers underestimate the importance of these recommendations. Perhaps they realize that they are not easily achieved. The first conversations that children have are not immediately productive or clear. Building a classroom environment in which children have the confidence to share their developing ideas about our number system takes time. The payoff, however, comes when a child such as Alejandro, who once thought that both the ones in eleven were tens, shares a strategy for the problem 34 + 48 + 21: 20 + 30 = 50, 50 + 40 = 90, 4 + 1 = 5, 5 + 8 = 13, 90 + 10 = 100, 100 + 3 = 103. Alejandro confidently communicates his competency with number and operations in a manner that *Principles and Standards* envisions.

Reference

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

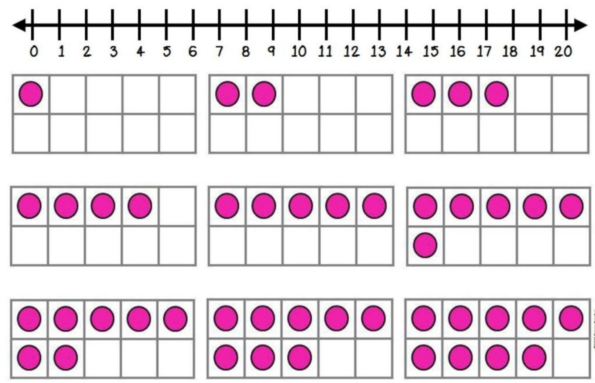
From our Vice-President Elementary

My name is Amy Johnston and I am serving as the KATM Elementary Vice President. I want to take a bit of your time to introduce myself. I have my undergraduate degree from Emporia State University with a focus in Early Childhood Education. I am in my sixteenth year at Auburn Elementary which is part of the Auburn Washburn school district. I have had the opportunity to teach preschool, kindergarten and am currently teaching second grade. I serve on our district math curriculum committee and enjoy sharing new ideas with other teachers. I live outside the small town of Dover with my husband and three children that are in 5 th , 3 rd , and 1 st grade. I enjoy "practicing" my ideas on them to get a feel for how my students will react!

Being a primary teacher, there is always a lot of discussion about Number Sense and how to know if a student has a true understanding of numbers or is just regurgitating memorized information. If you teach in the primary grades, you have probably felt like you spend FOREVER on numbers. I know my kindergarten colleagues have reflected over their curriculum and sometimes feel like they have taught the number four, for example, in every imaginable way and for some of our little ones it is still not concrete for them. What I have noticed is by second grade, when we start working more with two and three digit numbers, it becomes very clear to me who does and does not have an understanding of what numbers represent. Please understand I am not saying my Kindergarten and First grade teachers have not taught them well, but just that some students struggle to solidify the concept of numbers more than others. Some of my students spend a lot of time with me using base ten blocks as we practice making numbers, adding numbers, and taking away numbers using manipulatives. It seems as long as they can represent it with manipulatives, we are good, but no matter how much practice and talking we do, I take the manipulatives away and we are back to square one. (continued on next page)

So what is a person to do? First, take a deep breath. Then, find various ways to have students work with numbers. Some of my favorites include:

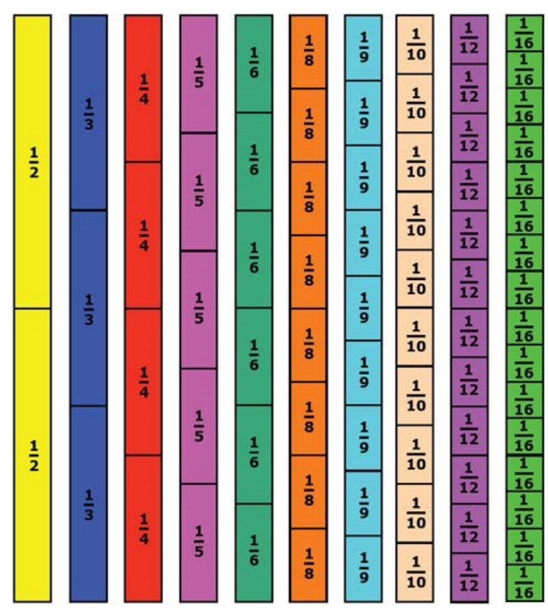
- Number of the Day—I have seen many Number of the Day activities. In second grade, I like to have students first work with 2 digit numbers and then work up to three digit numbers. Be careful not to make this jump too quickly as you will only have to go back to two digits if done too soon. Don't be afraid to have some students move to three digits while you continue on two digit numbers with others. Some of the skills that can be worked on during Number of the Day include one more, one less, ten more, ten less, draw base ten pictures to represent the Number of the Day, and value of digits.
- Place value card or dice games—Kids love to play games! One of the best resources for card and dice games, in my opinion, is from "Box Cars & One Eyed Jack," but these can also be found on Pinterest or teacher created! One game my students love to play is "Highest Wins" (also known as War).
- Making various tools available—If you want your students to have solid number sense, you want them to use manipulatives. In second grade some favorites include: double ten frames to be sure students understand how to count on from ten, base ten blocks to have discussions about the value of digits in a number and a hundred chart to find patterns in numbers. I have seen intermediate grades use fraction pies or bars to teach fractions. I have even seen 4th grade pull out the pattern blocks to show that 1 green triangle is 1/3 of the red trapezoid!



Click here to get a copy of this ten frame!

I know it may seem like developing number sense is the job of our primary teachers, but I would bet most of you reading this already have a student in your class that doesn't have solid number sense no matter what grade you teach. I encourage primary teachers reading this to not rush through teaching numbers. Intermediate teachers, if you have a student struggling with understanding numbers, take a walk down to your primary teacher friends and borrow some pattern blocks and base ten blocks.

While it is easy to just push through the curriculum, take time to be sure your students have strong number sense. Not only will your students benefit, but your middle school and high school colleagues will thank you!



Saving Money Using Proportional Reasoning

By Jessica A. de la Cruz and Sandra Garney

How would your middle school students solve this missing value problem:

> If 4 pounds of potatoes cost $6.00, how much would 10 pounds of potatoes cost?

Would they be more likely to apply the cross-multiplication algorithm, as opposed to simpler multiplicative reasoning approaches? Although cross multiplication results in a correct answer, students using this method do not necessarily understand proportionality. Rather than the more commonly used missing-value problems, like the previous example, we suggest posing comparison problems to help students recognize the underlying multiplicative relationship that exists within a proportion.

Consider this comparison problem:

> Which is a better price for potatoes: $1.29 for 10 pounds or $4.99 for 20 pounds?

Here, a factor-of-change strategy is intuitive (if the number of pounds doubles, then so should the price), and it emphasizes the multiplicative relationship between the two ratios. Moreover, cross multiplication is likely to be unsuccessful in determining the better price because students must interpret the relationship between the cross products. **Figure 1** presents an example of cross multiplication when applied to compare ratios.

Fig. 1 Using a cross-multiplication strategy to compare two prices yields two products that are difficult to interpret in terms of the original scenario.

$$\frac{\$2.50}{2\text{ lb.}} \stackrel{?}{=} \frac{\$3.25}{3\text{ lb.}}$$

$$(\$2.50)\times(3\text{ lb.}) \stackrel{?}{=} (\$3.25)\times(2\text{ lb.})$$

Many teachers would agree that once cross multiplication is introduced, their students tend to apply it by rote, abandoning all previously learned proportional reasoning strategies. Although cross multiplication is typically the most emphasized strategy in textbooks for solving missing-value proportion problems, many researchers believe that an overemphasis of this strategy is the root of students' difficulties with proportional reasoning. One study even found that students who were taught the cross multiplication strategy were actually less successful when solving proportion problems than students who were never taught the algorithm (Fleener, Westbrook, and Rogers 1993). Addi-

without knowledge of other proportional reasoning strategies is not enough to be considered proportional reasoning (Cramer, Post, and Currier, 1993; Fleener, Westbrook, and Rogers 1993). Students should fully develop more intuitive strategies, such as factor of change or unit rate strategies, before being introduced to cross multiplication. These intuitive strategies help students better understand the multiplicative relationship between proportional ratios. However, many textbooks heavily emphasize cross multiplication and leave a gap where teachers must develop other ways to foster the creation and use of different proportional reasoning strategies.

It is beneficial for students to discover intuitive strategies, as opposed to the teacher presenting strategies to them. Certain proportional reasoning tasks are more likely to elicit intuitive strategies than other tasks. The strategies that students are apt to use when approaching a task, as well as the likelihood of a student's success or failure solving it, are influenced by that task's context and numerical structure (de la Cruz 2013). Thus, teachers can encourage the development of particular strategies by carefully selecting the tasks that students will complete. Furthermore, implementing the Five Practices (Smith et al. 2009) can assist teachers in structuring the whole-class sharing of student-generated strategies in an organized and purposeful way. Considering the effects that task characteristics can have on strategy choices, we designed the Better Buy Lesson, which we describe here.

THE FIVE PRACTICES MODEL

Smith and colleagues (2009) present a model to support and prepare teachers to incorporate students' thinking into classroom discussion. Focusing on the Five Practices helps teachers by limiting the in-the-moment decisions that are sometimes frightening aspects of student-centered teaching. Moreover, it better prepares teachers to highlight the facets of students' thinking that tie specifically to the instructional goal. The Five Practices include the following: (1) anticipating, (2) monitoring, (3) selecting, (4) sequencing, and (5) connecting. When planning the Better Buy Lesson, we chose challenging mathematical tasks while anticipating the strategies that students would use when solving. Next, we selected the strategies we aimed to share in the discussion portion of the lesson by considering our ultimate instructional goals. Then we predicted how we would sequence the shared strategies, with the understanding that this sequence may be adapted, depending on what we observed when monitoring the classwork. Finally, we planned how we would connect the shared strategies to each other and to our instructional goals.

THE BETTER BUY ACTIVITY

Students, working in pairs, were asked to determine the better deal when given two different prices and quantities for similar items found in competing grocery store ads. They were instructed to use any strategy that they could fully explain to the class. In total, there were four comparison tasks (see **figs. 2–5**). Before the lesson, we chose each task carefully, anticipating strategies we thought students would use and after analyzing each task's numerical structure.

Choosing the Comparison Tasks and Anticipating Strategies

First, we predicted that students would have little success applying cross multiplication to compare the ratios, which is consistent with Singh's (2000) research. When the rates being compared are not proportionally related, interpreting the cross products is difficult (see **fig. 1**). It is clear from the unequal cross products that the ratios are not equivalent; however, it is not clear which one is the better buy. This meant that students would likely employ alternative strategies.

Second, we had two goals in mind when analyzing and selecting the four comparisons: To encourage flexible use of several proportional reasoning strategies and to emphasize the multiplicative nature of proportional ratios. Depending on the strategy, we chose particular numerical structures known to influence different problem-solving approaches (Tjoe and de la Torre 2013)

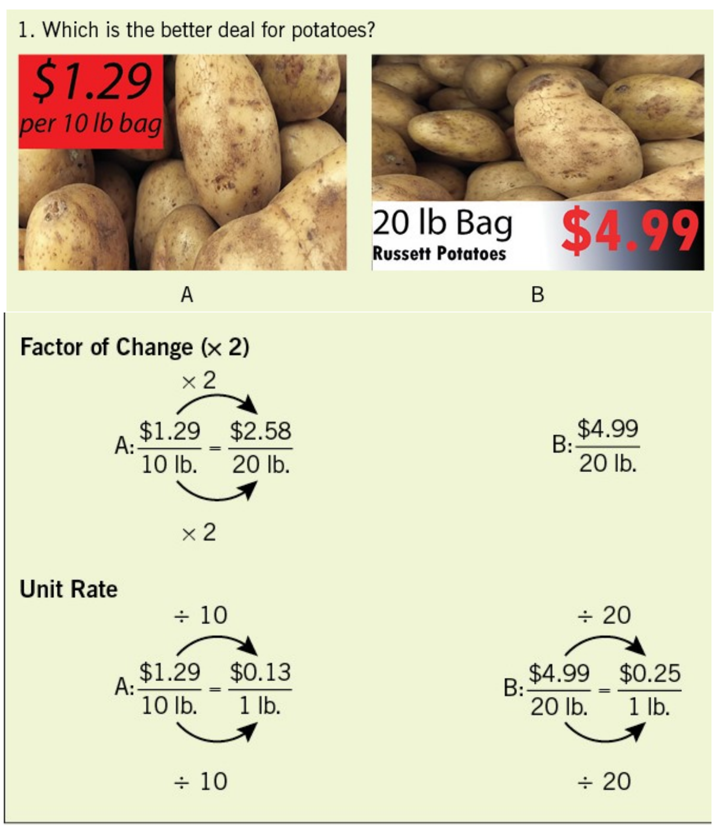
According to Lesh, Behr, and Post (1987), the presence of an integer factor of change between the ratios increases the likelihood that students would apply a factor of change strategy, also referred to in the literature as a building up through multiplication strategy (Steinthorsdottir and Sriraman 2009). The following comparison would likely encourage the use of a factor of change strategy: $15.00 for 4 pounds of dog food at store A versus $78.00 for 24 pounds at store B. At store A, we can determine the price for 24 pounds using a factor of change of 6:

$$\frac{\$15.00}{4\text{ lb.}} = \frac{\$90.00}{24\text{ lb.}}$$

The presence of an integer factor of change within one of the ratios (i.e., an integer unit rate), coupled with the absence of an integer factor of change between the ratios, encourages students to apply unit rate strategies. For instance, $15.00 for 5 pounds of dog food at store A versus $76.00 for 19 pounds would likely be solved using a unit rate strategy: At Store A, At Store B:

$$\frac{\$15.00}{5\text{ lb.}} = \frac{\$3.00}{1\text{ lb.}}; \qquad \frac{\$76.00}{19\text{ lb.}} = \frac{\$4.00}{1\text{ lb.}}.$$

Figure 2



Figures 2–5 present the four comparison tasks we created, highlight the numerical structure for each, and list the approaches that we anticipated students would use. When designing the activity, we aimed to have students perform these strategies: factor of change, unit rate, common denominator, and combination strategies. We looked for these specific strategies when we monitored the activity.

Monitoring Students' Work

According to Smith et al. (2009), teachers should monitor their students' thinking and strategies as they work to productively determine who should share and what should be shared in class discussion. Without careful monitoring and selecting, the discussion can turn into a "show and tell" of disconnected strategies and may not deepen students' understandings. **Figure 6** depicts the table we used to record our assessments throughout the monitoring process. It also indicates decisions that resulted when we anticipated students' approaches while also considering our instructional goal. We included an additional row at the bottom of the table to capture any unforeseen strategies as well as note incorrect additive approaches.

Figure 3

2. Which is the better deal for potatoes?

A B

Unit Rate

A: $\frac{\$1.29}{10\text{ lb.}} = \frac{\$0.13}{1\text{ lb.}}$ (÷ 10 applied to numerator and denominator)

B: $\frac{\$0.69}{1\text{ lb.}}$

Factor of Change (× 10)

A: $\frac{\$1.29}{10\text{ lb.}}$

B: $\frac{\$0.69}{1\text{ lb.}} = \frac{\$6.90}{10\text{ lb.}}$ (× 10 applied to numerator and denominator)

Figure 4

3. Which is the better deal for 12 packs of Coca-Cola?

A B

Unit Rate

A: $\frac{\$9.00}{4\text{ packs}} = \frac{\$2.25}{1\text{ pack}}$ (÷ 4)

B: $\frac{\$22.50}{10\text{ packs}} = \frac{\$2.25}{1\text{ pack}}$ (÷ 10)

Common Denominator

A: $\frac{\$9.00}{4\text{ packs}} = \frac{\$45.00}{20\text{ packs}}$ (× 5)

B: $\frac{\$22.50}{10\text{ packs}} = \frac{\$45.00}{20\text{ packs}}$ (× 2)

Combination of Buildup and Reduction

A: $\frac{\$9.00}{4\text{ packs}} = \frac{\$18.00}{8\text{ packs}}$ (× 2)

B: $\frac{\$22.50}{10\text{ packs}}$

$\frac{\$9.00}{4\text{ packs}} = \frac{\$4.50}{2\text{ packs}}$ (÷ 2)

$$\frac{\$9.00}{4\text{ packs}} = \frac{\$18 + \$4.50}{8 + 2\text{ packs}} = \frac{\$22.50}{10\text{ packs}}$$

Selecting, Sequencing, and Connecting Students' Work

After monitoring the students' work on the four comparison tasks and referencing our monitoring tool, specific groups were selected to share their strategies with the class. A pair who used long division to calculate the unit prices per pound of potatoes, in problem number 1, was asked to share first. Their work is depicted in **figure 7a**. Next, a pair was chosen to share their factor-of-change strategy (see **fig. 7b**). This strategy was presented after the unit rate strategy to illustrate the simplicity of the computations involved, in contrast to the previous method. Thus, the first comparison task led to a discussion of student-generated unit rate and factor of change strategies and motivated students to consider when one strategy would be more easily applied than another. Additionally, the teacher seized the opportunity to point out that a multiplicative relationship between ratios, as shown in the factor of change strategy, always exists when ratios are proportional. Further, the class discussed how the unit rate strategy is similar to a factor of change strategy. **Figure 8** illustrates how we find the unit price for potatoes at store A by multiplying the provided ratio by a factor of one-tenth, or divide by ten, to get a unit in the denominator.

Figure 5

If someone in our class had used an additive approach to compare these ratios, we would have addressed it by connecting to the context. For instance, if someone had explained that they added 10 pounds to get from 10 pounds to 20 pounds, so they also added $10.00 to the cost to get $11.29, we would have directed the class to notice that this would mean that the first 10 pounds cost $1.29, but the second 10 pounds cost $10.00. Since the cost for the same weight of potatoes should be the same, this additive strategy does not make sense.

Task 2 also involves potatoes; however, in this task one of the provided prices was given as a unit rate. The inclusion of a unit rate further encouraged the use of a unit rate strategy. This task was incorporated to ensure that a unit rate strategy would be shared. Unlike task 1, we determined the order in which the strategies would be presented while monitoring the classwork. To provide validation, we began with the most commonly used strategy. Again, the class discussed how both strategies, factor of change and unit rate, were related by looking at the multiplicative change involved in each.

Figure 6: A monitoring tool used to select and sequence the whole-class discussion.

	Strategy	Who and What	Order
Task 1	Unit Rate		First
	Factor of Change		Second
Task 2	Unit Rate		TBD
	Factor of Change		TBD
Task 3	Unit Rate		First (or omit)
	Common Denominator		Second
	Combination		Third
Task 4	Unit Rate		First (or omit)
	Common Denominator		Second
	Reduction		Third
Task ___	Other		

In task 3, students compared the prices for differing numbers of 12 packs of soft drinks, $9.00 for 4 at store A versus $22.50 for 10 at store B. This numerical structure is unique from the previous two tasks in that it involves equivalent ratios and the factor of change between ratios is not an integer. The aim of this task was

To elicit a common denominator strategy (e.g., find the cost of 20 or 40 of the 12 packs at each store) and a combination strategy (e.g., find the cost of 10 of the 12 packs at each store, by finding the cost of 8 and 2 of the 12 packs at store A and combining). **Figure 9** portrays the combination strategy that one group shared.

Figure 7

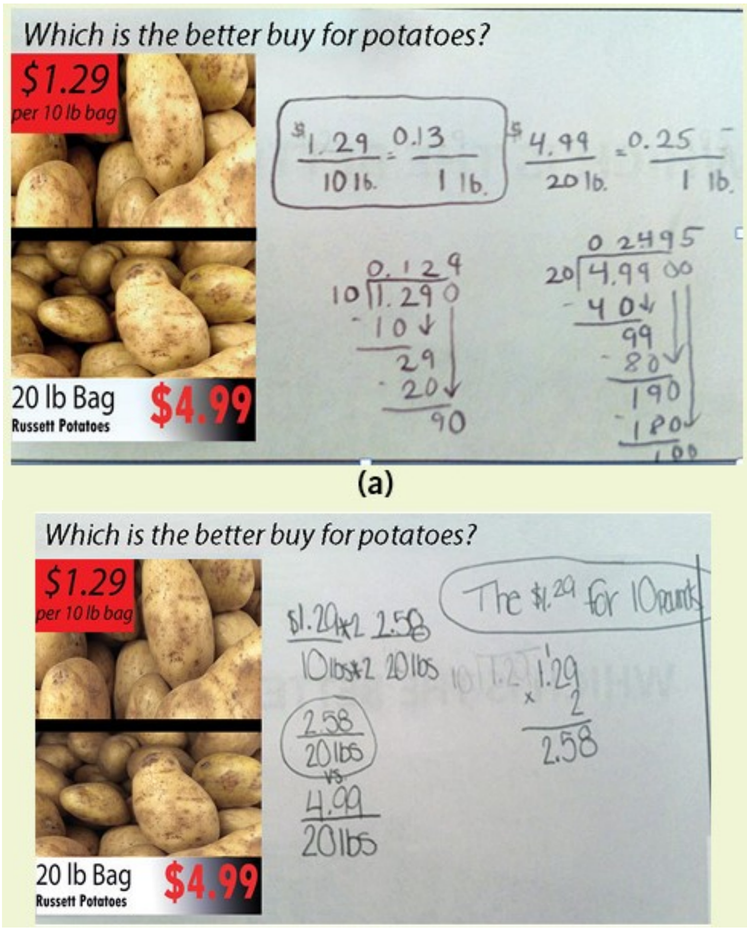
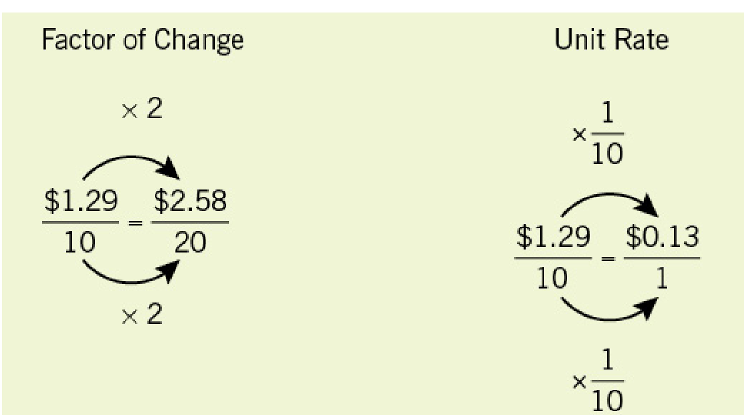


Figure 8



In the class discussion, we selected two groups who had used the two intended strategies to present their processes for the class. We sequenced the strategies in order of sophistication, with the common denominator strategy presented first. This strategy was deemed less sophisticated because it connected to the students' prior knowledge regarding fraction equivalence. According to Smith et al. (2009), it can be beneficial to begin with a strategy that is more familiar to students to validate their thinking and allow for connections between prior knowledge (equivalent fractions) and new knowledge (equivalent ratios). The class then discussed the similarities and differences between the common denominator strategy just witnessed and the factor of change strategies seen for task 1 and 2. Together we recognized that the common denominator strategy is a factor of change strategy where the common denominator found is not equal to either of the original denominators.

Second, we asked a group of students to explain their combination strategy, which was discovered by few students in the class. We then explicitly connected the students' work to the factor of change and reduction strategies discussed earlier by labeling each step according to the strategy it matched and labeling the multiplicative relationships with arrow diagrams, as shown in **figure 10**. We represented this combination strategy again, but more concretely, in a table (see **fig. 11**). Within the table, we used arrows to mark the multiplicative relationships. We chose to label the reduction from 4 to 2 as division by 2, as opposed to multiplication by 1/2 , because our students are more comfortable operating with whole numbers; however, we asked the class what our factor of change would be if we were to think of it as multiplication instead of division, to reiterate that a factor of change always exists.

Figure 9

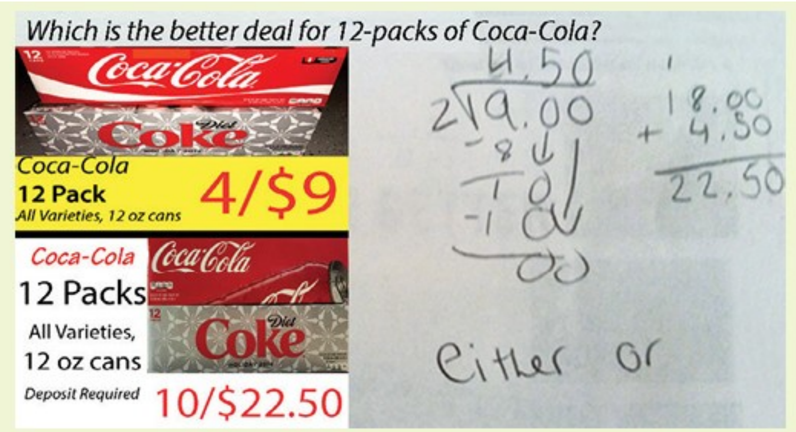


Figure 10

Step 1:
Factor of Change Strategy:

$$\frac{\$9.00}{4\text{ lb.}} = \frac{x}{8\text{ lb.}},\ x = \$18.00 \Rightarrow \$18.00 \text{ for 8 lb.} \quad (\times 2)$$

Step 2:
Reduction Strategy:

$$\frac{\$9.00}{4\text{ lb.}} = \frac{y}{2\text{ lb.}},\ y = \$4.50 \Rightarrow \$4.50 \text{ for 2 lb.} \quad (\div 2)$$

Step 3: Combine:

$$8\text{ lb.} + 2\text{ lb.} = 10\text{ lb.}$$
$$\$18.00 + \$4.50 = \$22.50 \Rightarrow \$22.50 \text{ for 10 lb.}$$

Figure 11

Number of 12 Packs	Cost ($) for 4 12-packs for $9.00
2 (÷ 2)	4.50 (÷ 2)
4	9.00
8 (× 2)	18.00 (× 2)
10 (2 + 8 = 10)	22.50 (4.50 + 18.00 = 22.50)

The final task asked students to compare three different deals for paper towels: 8 rolls for $8.99, 6 rolls for $7.99, and 2 rolls for $3.00. This task appeared last because there were three ratios to compare. The majority of our students used a unit rate strategy to compare the three ratios and, hence, the unit rate strategy was shared first. Next, when monitoring the groups as they worked, we noticed a reduction strategy, determining the price for 2 rolls according to each deal, and a common denominator strategy, finding the price for 24 rolls using each deal. Those groups were asked to detail their approaches for the class. We again connected these approaches to the ones shared earlier in the discussion by depicting, with arrows, the factor of change for each. We also asked, "Could we have used a factor of change strategy to find the price for 10 rolls?" Students then realized that the factor of change was 2.5, which we related to the combination strategy in which we found the price for 2 groups of 4 rolls and for 1/2 group of 4 rolls. We reiterated that the strategies that we discussed (factor of change, unit rate, common denominator, and combination) were all related to the factor of change strategy because equal ratios always have a multiplicative relationship that can be represented with an arrow diagram. Using the Five Practices and our well-thought-out tasks enabled us to effectively facilitate this student-centered lesson while achieving our content goals.

THE END RESULT: QUANTITATIVE REASONING

The Better Buy lesson not only provided an interesting and real-life context for studying proportional reasoning strategies but also required students to reason quantitatively and model with mathematics, two of the mathematical practices delineated within the Common Core State Standards for Mathematics (CCSSI

2010, pp. 6–8). Although this activity was used with an eighth-grade class to review and highlight the multiplicative structure of proportional situations, it is best suited for sixth-grade and seventh-grade audiences before cross multiplication and other proportional reasoning strategies are formally introduced. The students were so engaged in this activity that many groups finished the four assigned tasks and continued on to complete other grocery price comparisons.

Using the Five Practices model during the planning and implementation of this lesson in the classroom, we were able to effectively highlight multiple proportional reasoning strategies and their multiplicative properties while maintaining the student-centered aspect of our instruction. Allowing the students to generate their own methods for comparing the ratios based on their prior knowledge and intuitions enabled us to connect the formal ideas to their informal ones and, in turn, will lead to deeper understandings (de la Torre et al. 2013) of proportionality.

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Technology-Based Geometry Activities for Teaching Vector Operations

Although linear algebra has been included in the high school curriculum (CCSSI 2010), better support is needed for teachers. Most textbooks are written for college students and emphasize heavy computations, algorithms, and procedures. A newspaper article (Mathews 2012) described a seasoned high school teacher struggling with teaching linear algebra, making mistakes, being confused, and eventually quitting his job mid-semester. The school administration scrambled to find a replacement, going from substitute teachers to mathematics graduate students (from a nearby university) to its own mathematics chair, who had no previous experience teaching linear algebra either.

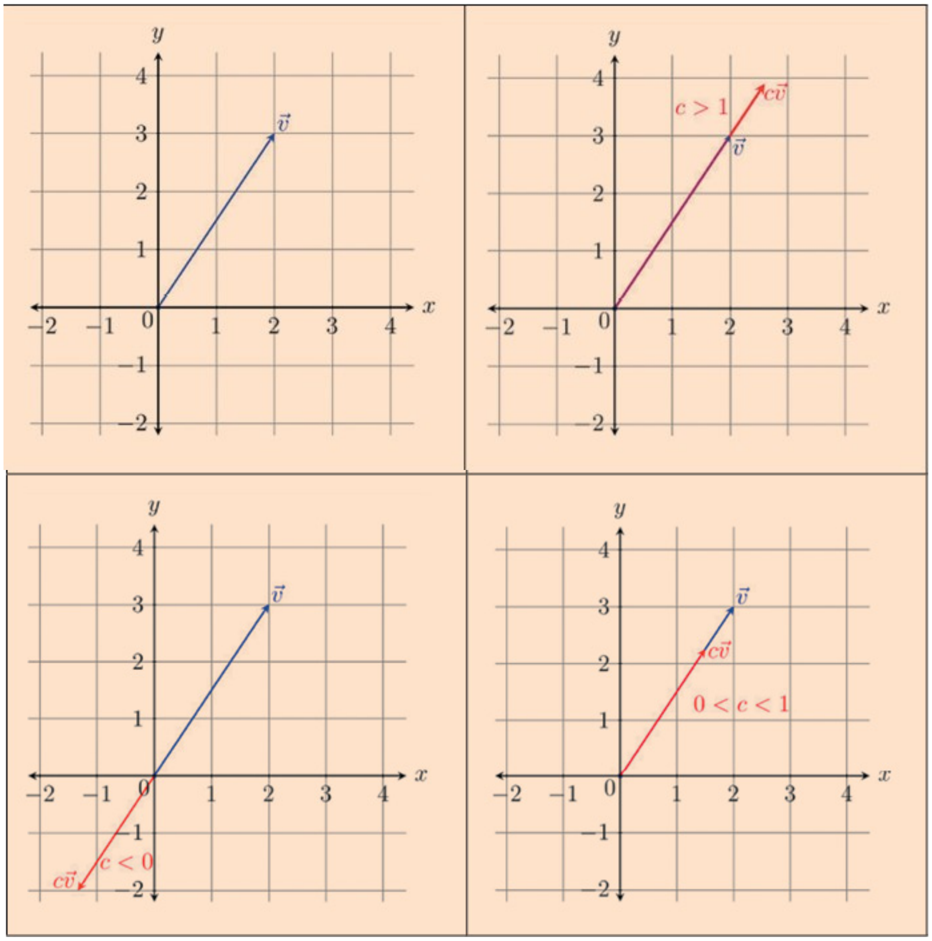
To address the need for teacher support and effective learning experiences for students, we present four activities that are specifically focused on making linear algebra intuitive, engaging, and hands-on. The topics are included in the vector and matrix quantities strand of the Common Core State Standards for Mathematics (CCSSI 2010):

- Recognize vector quantities as having magnitude and direction (CCSSI. MATH.HSN.VM.A.1)
- Addition and subtraction of vectors (CCSSI.MATH.HSN.VM.B.4)
- Scalar multiplication (CCSSI.MATH. HSN.VM.B.5)

We used The Geometer's Sketchpad® (GSP), although GeoGebra could also be used. Students working on these activities will need interactive geometry software skills for constructions including measurements, parameters, and calculations. We encourage teachers to implement all four activities, which took about seventy minutes, in this order, particularly because students working on them have demonstrated improved learning and understanding of vector operations (see Appova and Berezovski 2013). Discussion questions are included to encourage students to reflect on their observations.

ACTIVITY 1: SCALAR MULTIPLICATION

Defined as a directed line segment, a vector with its initial point (or tail) at the origin can be represented by the coordinates of the segment's endpoint (or tip). Thus, calculations in this activity used point coordinates to represent a vector, v. Students manipulated a parameter, c, to create a dynamic segment, cv. With attention to the cases $c > 1$, $c < 0$, and $0 < c < 1$, students recorded the scalar values c and noted the corresponding effect on the vector cv. The purpose was for the students to make explicit connections between the geometric representation of vectors and multiplication by a scalar as they noticed expansions, compressions, and changes in orientation. (See fig. 1.)



Discussion Questions

- What scalar c will make the directions of the vectors v and cv different? Explain why this occurs.
- What scalar c will make the sizes of the vectors v and cv different but will keep their directions the same? Explain.
- On the basis of your observations, identify the vectors cv with the same size as v.
- Describe all the vectors cv with the same direction as v.

Figure 1

ACTIVITY 2: VECTORS IN DILATIONS AND CONTRACTIONS OF SHAPES

This transitional activity helped strengthen students' ideas about scalar multiplication as they generalized from line segments to geometric shapes. By doubling and halving the vertex coordinates of a given rectangle, students directly observed and explained the meaning of dilation and contraction as the stretching and shrinking of geometric shapes.

To begin, students constructed rectangle A1B1C1D1 with coordinates that are obtained from the coordinates of ABCD by scalar multiplication (see fig. 2). Students examined the sizes of the rectangles and considered the proportional changes and scale factors of the sides. Stretching or shrinking the original (blue) rectangle resulted in the similar scaled (green and red) objects.

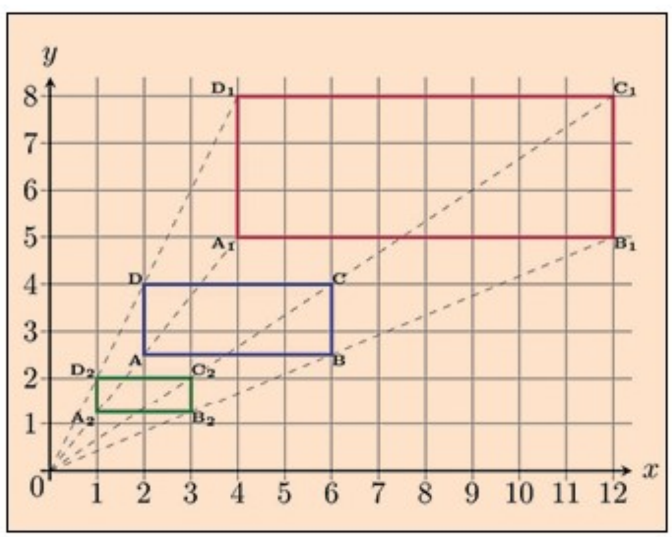


Fig. 2 A screen shot shows dilations and contractions of the blue rectangle.

Discussion Questions

- Construct a rectangle A1B1C1D1 by doubling the coordinates of rectangle ABCD. Measure the lengths of the sides of the original rectangle and the new rectangle. What do you notice?
- Explain the relationship between the corresponding vertices of rectangles ABCD and A1B1C1D1 in terms of vectors.
- Construct the rectangle A2B2C2D2 by halving the coordinates of ABCD. How are the three rectangles that you have created related? How does the relationship among these rectangles relate to vectors?
- In your own words, explain the meaning of a dilation and a contraction.

ACTIVITY 3: VECTOR ADDITION

This activity targeted students' understanding of vector addition through geometric constructions, manipulations, and observations using technology. The software allowed students to move the vectors u, v, v + u and u + v around the screen while tracking the coordinates (see fig. 3). Using technology, students explored and answered construction-based questions.

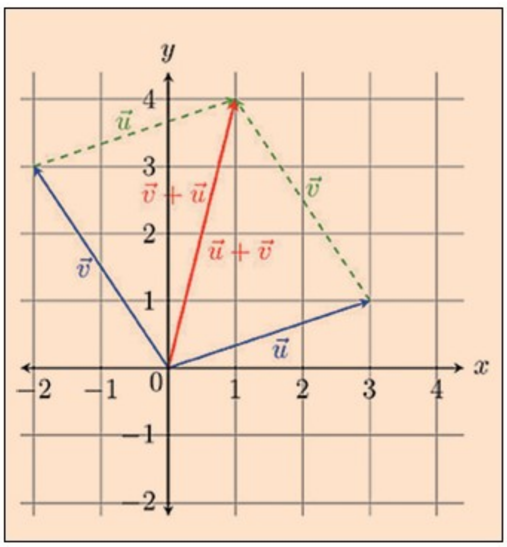


Fig. 3 The vector addition activity explores commutativity using the triangle and parallelogram constructions.

From class, many students were aware of the coordinate addition rule. However, they also needed to be able to use the software to geometrically construct the resultant vector v + u or u + v, given vectors u and v. Geometric interpretations (i.e., the triangle and parallelogram rules) helped students observe and explain why vector addition is commutative. Some students identified the parallelogram as being composed of two congruent triangles; some noted that opposite sides of a parallelogram are congruent. Discussion led students to recognize that the decomposition of a vector into a sum of two vectors is not unique. (See fig. 4.)

As part of this activity, teachers might ask students to complete the following tasks:

- Using the coordinate addition rule, add vectors u and v to construct the resultant vector u + v.
- Construct the vectors v + u (by adding u to the tip of v) and u + v (by adding v to the tip of u). Compare the vectors v + u and u + v, with the vector that you constructed using the coordinate addition rule.

Discussion Questions

- Explain why the different constructions adding u and v result in the same vector.
- Change the magnitudes of u and v to check that commutativity still holds. In your own words, explain why addition of vectors is commutative.

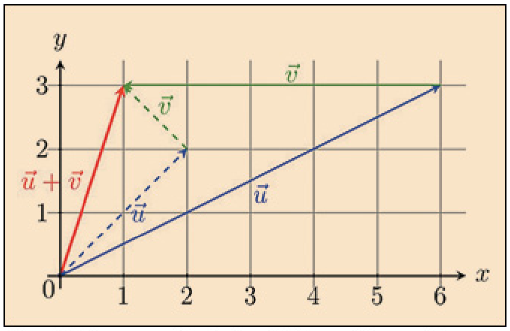


Fig. 4 The resultant $\vec{u} + \vec{v}$ (in red) may be longer than both $\vec{u}$ and $\vec{v}$ (shown as dashed segments) or shorter (shown with solid segments).

- Is it possible to find other pairs of vectors (different from the given u and v) that will add up to the same resultant? If so, use technology to construct an example. If not, explain why not.
- Is it possible to find a pair of vectors u and v so that u + v is shorter than u and shorter than v? If so, use technology to construct an example. If not, explain why not.
- Is it possible to find a pair of vectors u and v so that u + v is longer than u and longer than v? If so, use technology to construct an example. If not, explain why not.

ACTIVITY 4: VECTOR SUBTRACTION

In this activity, students applied knowledge from the previous activities: scalar multiplication, construction of opposite vectors, and vector addition. They considered vector magnitude, direction and coordinates. Students made specific connections between a geometric representation of the triangle or parallelogram rule and the addition or subtraction of vectors (see fig 5). Some students noted that subtracting a vector is the same as adding its opposite. In these explorations, the software allowed students to move the vectors but keep their magnitude and direction fixed (unchanged). Students could be asked to complete the following tasks.

- Construct a vector w starting at the terminal point of v and connecting to the terminal point of u. Provide its coordinates and magnitude.
- Construct a vector s, which is the sum of u and −v. Provide its coordinates and magnitude.

Discussion Questions

- How do u, v and w relate in terms of their coordinates and magnitude?
- How do w and s relate in terms of their coordinates and magnitude?

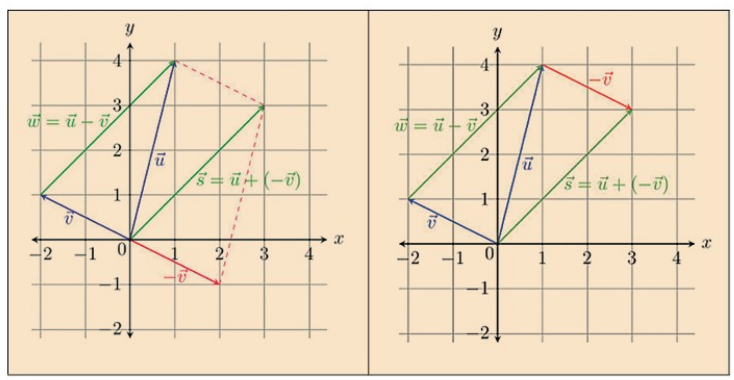


Fig. 5 Sample screen shots show vector subtraction.

- Given another set of two vectors u and v, explain how you would create a vector w, such that w = u—v.

GEOMETRIC INTERPRETATION IN LINEAR ALGEBRA

Research studies strongly emphasize the use of geometric structures and representations in linear algebra (Harel 1989; Tabaghi 2010; Gueudet-Chartier 2002). These four activities target students' learning through geometric interpretations of vectors (Tabaghi 2010), the development of students' concept images (Harel 1989), and geometric intuitions (Gueudet-Chartier 2002). The discussion questions are included to help teachers situate linear algebra in a more engaged and hands-on learning environment in their classrooms. As an additional resource, a Geometer's Sketchpad file is available with the online article.

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CALL FOR SUBMISSIONS

Your chance to publish and share your best ideas!

The KATM Bulletin needs submissions from K-12 teachers highlighting the mathematical practices listed above. Submissions could be any of the following:

- Lesson plans
- Classroom management tips
- Books reviews
- Classroom games
- Reviews of recently adopted resources
- Good problems for classroom use
-

Email your submissions to our Bulletin editor: jennywilcox@katm.org

Acceptable formats for submissions: Microsoft Word document, Google doc, or PDF.

Call for Nominations!!!

The KATM Board is currently taking nominations to fill the following positions in the upcoming Board election. We are looking for educators that are interested in taking a leadership role in the field of Math Education throughout the State of Kansas. You can nominate yourself or someone that you know that has demonstrated a passion for advancing math in our state as well as someone that has a lot to offer in the way of supporting teachers. Please email Fred Hollingshead, Past President (hollingsheadf@usd450.net), with nominations and contact info of the nominee or fill out the online nomination form found at katm.org. Regular members in good standing are eligible for positions on the KATM Board. Nominations need to be completed by February. Elections will be held online at www.katm.org in March. A notice will be sent to remind you to vote.

Positions available for the upcoming election:

President-elect * 4-year term

The president-elect will serve for one year before then becoming president for a year, and then past-president for two years. The president-elect will assume the duties of president when needed. As president, the elected individual will preside over all KATM events and business meetings. The president will conduct the business of KATM as directed by the Executive Board and will represent KATM at a variety of functions, meetings, and conferences. The president is responsible for the overall functioning of the organization with assistance from the officers and Board members. As the past-president taking office in even-numbered years, this position will serve as the community relations representative for 2 years. This person shall be responsible for assuring communication between the Association and legislative, executive, and administrative branches of the government of Kansas.

Vice President – College * 2-year term

The vice president for college will attend all meetings and conferences and will assist the president in conducting the business of KATM. Each vice president will also encourage membership and will promote issues of special interest to their level represented in addition to serving on various committees as assigned. The person elected to this position will act as a liaison to college teachers.

Vice President – Middle School * 2-year term

The vice president for middle school will attend all meetings and conferences and will assist the president in conducting the business of KATM. Each vice president will also encourage membership and will promote issues of special interest to their level represented in addition to serving on various committees as assigned. The person elected to this position will act as a liaison to middle school teachers.

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KATM Cecile Beougher Scholarship
ONLY FOR ELEMENTARY TEACHERS!!

A scholarship in memory of Cecile Beougher will to be awarded to a practicing Kansas elementary (K-6) teacher for professional development in mathematics, mathematics education, and/or mathematics materials needed in the classroom. This could include attendance at a local, regional, national, state, or online conference/workshop; enrollment fees for course work, and/or math related classroom materials/supplies.

The value of the scholarship upon selection is up to $1000:

- To defray the costs of registration fees, substitute costs, tuition, books etc.,
- For reimbursement of purchase of mathematics materials/supplies for the classroom

An itemized request for funds is required. (for clarity)

REQUIREMENTS:

The successful candidate will meet the following criteria:

- Have a continuing contract for the next school year as a practicing Kansas elementary (K-6) teacher.
- Current member of KATM (if you are not a member, you may join by going to www.katm.org. The cost of a one-year membership is $15)

APPLICATION:

To be considered for this scholarship, the applicant needs to submit the following no later than June 1 of the current year:

1. A letter from the applicant addressing the following: a reflection on how the conference, workshop, or course will help your teaching, being specific about the when and what of the session, and how you plan to promote mathematics in the future.
2. Two letters of recommendation/support (one from an administrator and one from a colleague).
3. A budget outline of how the scholarship money will be spent.

Notification of status of the scholarship will be made by July 15 of the current year. Please plan to attend the KATM annual conference to receive your scholarship. Also, please plan to participate in the conference.

SUBMIT MATERIALS TO:

Betsy Wiens
2201 SE 53rd Street
Topeka, Kansas 66609

Go to www.katm.org for more guidance on this scholarship

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Capitol Federal Mathematics Teaching Enhancement Scholarship

Capitol Federal Savings and the Kansas Association of Teachers of Mathematics (KATM) have established a scholarship to be awarded to a practicing Kansas (K-12) teacher for the best mathematics teaching enhancement proposal. The scholarship is $1000 to be awarded at the annual KATM conference. The scholarship is competitive with the winning proposal determined by the Executive Council of KATM.

PROPOSAL GUIDELINES:

The winning proposal will be the best plan submitted involving the enhancement of mathematics teaching. Proposals may include, but are not limited to, continuing mathematics education, conference or workshop attendance, or any other improvement of mathematics teaching opportunity. The 1-2 page typed proposal should include

- A complete description of the mathematics teaching opportunity you plan to embark upon.
- An outline of how the funds will be used.

An explanation of how this opportunity will enhance your teaching of mathematics.

REQUIREMENTS:

The successful applicant will meet the following criteria:

- Have a continuing contract for the next school year in a Kansas school.
- Teach mathematics during the current year.

Be present to accept the award at the annual KATM Conference.

APPLICATION:

To be considered for this scholarship, the applicant needs to submit the following no later than **June 1 of the current year.**

- A 1-2 page proposal as described above.

Two letters of recommendation, one from an administrator and one from a teaching colleague.

PLEASE SUBMIT MATERIALS TO:

Betsy Wiens, Phone: (785) 862-9433, 2201 SE 53rd Street, Topeka, Kansas, 66609

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KLFA Update

KLFA Hears the Importance of Early Childhood Education to Kindergarten Readiness

•June 10, 2016•

Kansas Learning First Alliance (KLFA) met June 10, 2016 at the Kansas National Education Association (KNEA) building. Dayna Richardson, KLFA Chair, started the meeting with a Legislative/Policy update.

Members were updated on legislative and policy issues, including information on school accreditation and Legislative Special Session. All members were encouraged to continue advocating for our students, our educators and our Kansas schools and to vote and encourage everyone they know to vote in the primary and general election.

Kathy Busch, State Board of Education, shared that the state accreditation model, **Kansas Education Systems Accreditation (KESA**) will be voted on at their next meeting. The group expressed the importance of focusing on all five Rs: Relationships, Relevance, Responsive Culture, Rigor, and Results.

Vera Stroup-Rentier, KSDE, Donna Booe, Kansas Children Service League, and Haley Pollack, Kansas Action for Children shared their respective work regarding the importance of **Early Childhood Education** and the challenges they face in terms of shrinking budgets. Key points from their presentations:

In the last five years 7,000 more kids have entered foster care, well above the national average.

Children's Initiative Fund (CIF), monies from a tobacco settlement, is used to promote the health and welfare of Kansas children. The Governor has suggested eliminating this funding which Ffocusesing on healthy development, strong families, and early learning. The Governor has suggested eliminating this funding.

Shared the plan to screen 4,000 students next year to establish a measurement of kindergarten readiness.

Members continued their work in one of three focus areas: **Professional Learning**, **Student Success**, and **Community Engagement**. The groups reflected on this year's work to outlined plan the work they will complete over the next year, elected Lori Laurie Curtis as chair elect, and presented Tom Krebs the **Karen Godfrey Leadership Award** for his exceptional work with KLFA for many years.

KATM Minutes: June

Installation of new officers: David Fernkopf: President ; Stacey Ryan: President-elect;
Pat Foster: Past-president – Community Relations; Lanee Young: Vice President of College;
Amy Johnston: Vice President of Elementary; Amber Hauptman: Vice President of High School;
Jerry Braun: Zone 1 Representative; Janet Stramel: Secretary; Zone 4 Representative - unfilled;

Discussion of Capitol Federal Scholarship and Cecile Beougher scholarship.

Treasurer's report

Editor's report and ideas for future of Bulletin: It was decided that we would continue to provide NCTM articles in our Bulletins as a service to our members. Ideas for articles include *successful lesson plans, interview a teacher, book reviews, website reviews, spotlight a math teacher, etc.*

KLFA report was given….see previous page.

Update on website

Update on fall conference: Chris Shore will be keynote speaker.

Update on standards revision committee: KATM members that have served on the Standards Revision committee reported that there are three subcommittees: Elementary, Middle and High School looking at the Kansas Standards. The committee is going through and clarifying each standard. There are two senators on the committee as well. The goal is for the revised standards to be presented to the Kansas Board of Education in the fall 2016 for implementation in fall 2017.

Next meeting: October 13, Maize Kansas

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NCTM Update:

My name is Stacey Bell and I am pleased to be the NCTM Rep for KATM. NCTM has a new website design and has been focusing on developing its Affiliate Site for its members. As an affiliate of NCTM, KATM is able to now post our upcoming events on this new site for neighboring states to see. And likewise, we are able to see what other affiliates are doing around us. You should check it out at http://www.nctm.org/affiliates/

In other news, the NCTM election is coming up. Voting starts Sept. 30 for NCTM members. There are five positions to vote for to fill their executive board of directors.

Candidates for President-Elect (one will be elected)

Robert Q. Berry III, University of Virginia, Charlottesville, VA

Nora G. Ramirez, Mathematics Consultant, Tempe, AZ

Candidates for Director, High School Level (one will be elected)

David Ebert, Oregon High School, Oregon, WI

Jason Slowbe, Great Oak High School, Temecula, CA

Candidates for Director, At-Large (three will be elected)

Vanessa Cleaver, Little Rock School District, Little Rock, AR

Linda Ruiz Davenport, Boston Public Schools, Boston, MA

Rick A. Hudson, University of Southern Indiana, Evansville, IN

DeAnn Huinker, University of Wisconsin Milwaukee, Milwaukee, WI

Daniel J. Teague, North Carolina School of Science and Mathematics, Durham, NC

Desha L. Williams, Kennesaw State University, Kennesaw, GA

To find out more about these candidates and make informed decisions go to:

http://www.nctm.org/election/ to find bios for each candidate. You will also want to make sure your email address is current.

Being members of both organizations allows you to have a wealth of resources for

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Do you like what you find in this Bulletin? Would you like to receive more Bulletins, as well as other benefits?

Consider becoming a member of KATM.

For just $15 a year, you can become a member of KATM and have the Bulletin e-mailed to you as soon as it becomes available. KATM publishes 4 Bulletins a year. In addition, as a KATM member, you can apply for two different $1000 scholarship.

Current members—-refer three new members and you get one free year of membership!

Join us today!!! Complete the form below and send it with your check payable to

KATM to:
Margie Hill
KATM-Membership
15735 Antioch Road
Overland Park, Kansas 66221

Name_______________________________

Address____________________________

City_______________________________

State______________________________

Zip________________________________

Home Phone_________________________

HOME or PERSONAL EMAIL:

____

Are you a member of NCTM? Yes___ No___

Position: (Cirlce only one)

Parent
Teacher:: Level(s)________
Dept. Chair
Supervisor
Other

Referred by: ______________________________

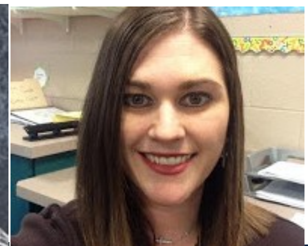
____

KANSAS ASSOCIATION MEMBERSHIPS

Individual Membership: $15/yr. ___
Three Years: $40 ___
Student Membership: $ 5/yr. ___
Institutional Membership: $25/yr. ___
Retired Teacher Membership: $ 5/yr. ___
First Year Teacher Membership:$5/yr. ___
Spousal Membership: $ 5/yr. ___
(open to spouses of current members who hold a regular Individual Membership in KATM)

KATM Executive Board Members

President: David Fernkopf, Principal, Overbrook Attendance Center, dferkopf at usd434.us



President: Elect: Stacey Ryan, Middle School teacher, Anodover Middle School, ryans@usd385.org

Past President, NCTM Rep: Stacey Bell, Instructional Coach, Shawnee Heights Middle School

bells at usd450.net, 785-379-5830

Past President, Community Relations: Pat Foster
Principal, Oskaloosa Elementary School
pfoster at usd341.org

Secretary: Janet Stramel, Assistant Professor, Fort Hays State Univ.

jkstramel at fhsu.edu

Vice President, College: Lanee Young

Membership Co-chairs: Margie Hill, Instructor, Kansas University

marghill at @ ku.edu

Vice President High School: Amber Hauptman, Math Teacher, Washburn Rural High School

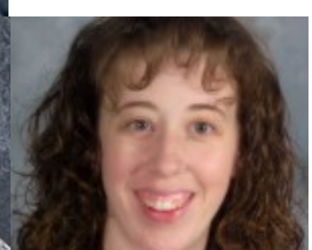
hauptamb at usd437.com

Membership Co-Chair: Betsy Wiens, Math Consultant

albf2201 at aol.com

Vice President Middle School: This position is currently unfilled.

Treasurer: David Fernkopf, Principal, Overbrook Attendance Center, dferkopf at usd434.us



Vice President Elementary: Amy Johnston, 2nd grade Teacher, Auburn Elementary

johnsamy at usd437.com

KSDE Liaison: Melissa Fast, Math Education Consultant
mfast at ksde.org

Bulletin Editor: Jenny Wilcox, 7th grade teacher, Washburn Rural Middle School,
wilcojen at usd437.net

KATM Executive Board Members

Zone 1 Coordinator:

Jerry Braun, Hays Middle School, jj_ks at yahoo.com

Zone 4 Coordinator:

Zone 2 Coordinator:

Kira Pearce

Zone 5 Coordinator:

Lisa Lajoie-Smith, Instructional Consultant, llajoie at sped618.org

Zone 3 Coordinator:

Zone 6 Coordinator:

Jeanett Moore, 2nd grade teacher, USD 48

Jeanett.moore at usd480.net

Webmaster: Fred Hollingshead Instructional Coach, Shawnee Heights High School hollingsheadf at usd450.net