Supporting opportunities for productive struggle: Implications for planning mathematics lessons

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IN MY OPINION

Ongoing dialogue about the most effective approach to teach mathematics to elementary school learners has been held in the context of new mathematics standards (e.g., the Common Core), which require students to develop more conceptual understanding of mathematics concepts. In many instances, these standards emphasize the importance of reasoning and problem solving; they include recommendations that students have opportunities to investigate mathematical tasks and situations, determine how to represent and solve these tasks, reason about their solution, and make connections between mathematics concepts.

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Research documents that students consistently struggle with how to approach, set up, solve, and reason about cognitively demanding mathematical tasks (U.S. DOE and IES 2015). Rigorous tasks require students to read a situation and determine how to find the answer. Recent recommendations from national leaders in mathematics education call for teachers to supply more opportunities for students to engage in productive struggle by solving word problems and challenging mathematical tasks (NCTM 2014) for which students draw on mathematics concepts to determine how to solve tasks (Carpenter et al. 2014).

The current dichotomy that exists is not a question of whether these types of tasks and rigorous problems should be posed but *when* in a lesson they should be posed. Many mathematics educators argue for starting the lesson with a task (e.g., Smith, Bill, and Hughes 2008); however, a number of mathematics curricular resources have been written using a Gradual Release model (e.g., Fisher

TABLE 1

In contrast to traditional models in which the entire class participates in a lesson regardless of students' math ability, with the 5E model, any direct teaching or teacher modeling is targeted in small groups and given only to students who truly need it.

Comparison of the Indirect Instruction (5E) model and the Gradual Release model

Indirect (E model)	Elements of the gradual-release model
Engage Students participate in a whole-class mathematical activity, such as a number talk, a problem of the day, or a classroom routine.	The teacher poses questions and checks for student understanding.
Explore Students are given time to work on the task with their partner or small group. The teacher gives instructions only and asks questions to support task exploration.	Students explore math in a hands-on manner. The teacher poses questions and checks for student understanding.
Explain Students come together to discuss the problem and ways they solved it. The teacher may select a main focus on the basis of observations; facilitates the discussion. The teacher may then instruct students as needed.	The class discusses concepts.
Extend Students continue to work on the concept throughout the remainder of the lesson with activities, math games, and small-group work. *This is the time that the teacher can address small groups for differentiated needs.*	I do/We do/You do in small groups. Instruction is differentiated for rigor or intervention. Students apply their knowledge and skills in activities and games.
Evaluate Students solve a final task or participate in a discussion of concepts. The teacher is able to evaluate student learning, which informs the planning for future lessons.	Students show what they know. The teacher evaluates student performance.

and Frey 2013), with which teachers first model ("I do") how to solve tasks and then slowly give responsibility of doing math to students during the course of the lesson ("We do" and "You do"). Although research from the special education field cites benefits to the idea of heavily guided instruction for students who have been identified with exceptional learning needs (e.g., Responsiveness to Instruction tier 3), research from the seminal Cognitvely Guided Instruction project (CGI) (Carpenter et al. 2014) found that using an instructional plan focused on having students first solve tasks *without* teacher modeling led to gains in problem solving and also increased student engagement in mathematics lessons (Carpenter et al. 2000). Further, on tests of computational skills, the CGI approach was just as effective as more teacher-directed approaches, such as the traditional Gradual Release model.

What this means for teachers today is that great potential exists for enhancing their students' mathematical understanding by having students explore cognitively demanding mathematical tasks. When a teacher models and provides direct instruction at the start of a lesson, it rarely enables students to explore mathematical tasks or engage in productive struggle (Munter, Stein, and Smith 2015).

One way to incorporate productive struggle is to deliberately plan phases or aspects of mathematics instruction. In mathematics, this has been written about as a before-during-after cycle (Burns 2015), or the 5E approach, which includes *Engage*, *Explore*, *Explain*, *Elaborate/Extend*, and *Evaluate* stages (Polly 2014). **Table 1** explains the features of each model as well as where the features of the Gradual Release model appear in each. With the 5E model, students spend a majority of mathematics lessons exploring mathematical tasks with support from the teacher. Further, any direct teaching or teacher modeling is targeted in small groups and given only to students who truly need it, as opposed to more traditional models in which the entire class participates in a lesson regardless of a student's mathematics ability.

As you continue to examine how mathematics instruction is taught at your school, focus on two questions: (1) How much time are students spending exploring mathematics concepts and doing mathematics, and (2) How is mathematics instruction modified to meet the needs of all students in the classroom? On the basis of what we know about how students learn mathematics, such Indirect Instruction models as the 5E model provide more opportunities for both of these questions than such traditional approaches as the Gradual Release model.

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COACHES' CORNER

Computational fluency

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In a document outlining key shifts of the Common Core State Standards, *rigor* comprises three attributes: (1) conceptual understanding, (2) procedural skills and computational fluency, and (3) application.

Fluency can be defined as "speed and accuracy in calculation." Automaticity in single-digit multiplication provides increased and improved access to concepts and procedures that are more complex. Strategic games motivate students to practice facts. In the Line Up game, students move one paper clip at a time along the factor row to create a multiplication fact. The resulting product is marked with a chip. Players are challenged to build a row of chips in a vertical, horizontal, or diagonal row. In identifying the best chip placement, students must consider several options, all requiring the product of two factors.

Supply players with a blank game board and a multiplication table as a reference. This will (a) enable students of all levels to compete, (b) provide visual reinforcement of facts, and (c) maximize the game's enjoyment.

Questions? Comments? Contact **robyn@robynsilbey.com.**

Line up! Multiplication

8	10	12	15	18
20	27	32	36	24
56	63	72	42	48
45	40	35	30	54
28	16	14	21	81

2	3	4	5	6	7	8	9

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Go to **http://www.nctm.org** to access three full-size activity sheets for "Coaches' Corner: Computational Fluency." Access to this online material is a members-only benefit.

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*In our sister journal**

mathematics teaching in the **MIDDLE SCHOOL**

TCM readers may want to look for “We’re in Math Class Playing Games, Not Playing Games in Math Class,” by P. Janelle McFeetors and Kylie Palfy in the May issue of *Mathematics Teaching in the Middle School*. The authors’ hope in integrating abstract strategy games in mathematics class was that students would develop ways of reasoning logically about the strategies they were creating. One of the surprises was how quickly students improved in their reasoning. This was heard as students’ comments became increasingly convincing statements and their game play became increasingly effective.