KATM Bulletin

Kansas Association of Teachers of Mathematics

2017 Fall Conference

Seaman Middle School in Topeka

October 16, 2017

Keynote Speaker: Andrew Stadel

Andrew Stadel is in his third year as a Math Instruction and Digital Learning Coach for Tustin Unified School District in California. Having taught secondary math for over ten years, he believes estimation is key to building number sense. On his widely-acclaimed Estimation 180, Andrew offers teachers free estimation challenges, lessons, and readymade resources to use tomorrow with their students. Andrew also shares his passion for student thinking and mathematical exploration by presenting at national conferences and as a consultant for school districts, supporting teachers by strengthening their instructional tool belts. When Andrew is not covering file cabinets with sticky notes, he enjoys spending time with his wife, building Lego vehicles with his son, and playing Disney princesses with his daughter.

Here is an example of another estimation task from Andrew's blog.

What will be the value of the dollar symbol of quarters?

Make an estim

* Required

What's too LOW? *

What's too HIGH? *

Your estimate. *

Your reasoning. *
Do better than "I guessed."

ESTIMATION180.COM

A Message from our President
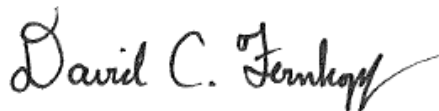
Hello Kansas Math Teachers!

In this issue of the KATM Bulletin we once again offer some great articles from NCTM. These articles focus on algebra and functions .

If you have not had time to look at the new draft math standards for Kansas, please take a moment and look at them. The link for the standards can be found on our KATM Facebook page. There are changes K-12 but the biggest changes can be found at the secondary level. You can give feedback online, or there are several meetings to give feedback.

Also, I want to encourage everyone to consider applying for the scholarships we have to offer each year. The first scholarship we have to offer is the KATM Cecile Beougher Scholarship. This scholarship is for elementary teachers. More information on the scholarship can be found on page 37. This scholarship is for up to $1000, and applying is easy! The second scholarship we offer is the Capitol Federal Mathematics Teaching Enhancement Scholarship. This scholarship is for K-12 teachers. More information on this scholarship can be found on page 38. Again, applicants can apply for up to $1000. Please consider applying for one or both of the scholarships!

Don't forget to mark your calendars for October 16, 2017 for the 2017 KATM conference. We will be in Topeka this year at Seaman Middle School. The conference planning committee is already hard at work, trying to bring you a conference to be excited about. Consider attending this great conference, or even being a presenter.

Take some time and see what information KATM has to share with you.

David C. Fernkopf

David C. Fernkopf

President, KATM

davidfernkopf@katm.org

Hello Kansas educators! I hope your year is going well as you have started your second semester. Things always seem to move quickly as we get to this second half of the year. We just had our January KATM meeting, and I always come back from these meetings energized with all of the ideas that KATM has for how to best serve Kansas educators! We would love to start featuring some of our members in our Bulletin and hear about the great things you're doing in your classroom. If you would like to be featured in an upcoming Bulletin, send me an email at jennywilcox@katm.org and I'll be in touch!

Jenny Wilcox

Jenny Wilcox
KATM Bulletin Editor

CALL TO PRESENTERS:

If you're interested in presenting at the 2017 KATM Conference, we want to have a wide variety of sessions available. You can submit a proposal for a conference session on the KATM website by clicking on the "Conference" tab.

CALL FOR SUBMISSIONS

Your chance to publish and share your best ideas!

The KATM Bulletin needs submissions from K-12 teachers highlighting the mathematical practices listed above. Submissions could be any of the following:

◊ Lesson plans
◊ Classroom management tips
◊ Books reviews
◊ Classroom games
◊ Reviews of recently adopted resources
◊ Good problems for classroom use
◊
Email your submissions to our Bulletin editor: jennywilcox@katm.org

Acceptable formats for submissions: Microsoft Word document, Google doc, or PDF.

Common Core State Standards Mathematics Standards Progressions

Kinder-garten	1	2	3	4	5	6	7	8	High School
Counting and Cardinality									Number and Quantity
Number and Operations in Base 10						Ratio and Proportional Relationships			
			Number and Operations: Fractions			The Number System			
Operations and Algebraic Thinking						Expressions and Equations			Algebra
								Functions	Functions
Geometry						Geometry			Geometry
Measurement and Data						Statistics and Probability			Statistic and Probability

Last year, KATM wrapped up our series on the mathematical practices. This year, we begin a new series, focused on the standards progressions. We will be focusing on how topics progress and change over the K-12 curriculum.

February 2017: Operations and Algebraic Thinking to Expressions and Equations/Functions to Algebra and Functions

April 2017: Geometry

October 2017: Measurement and Data to Statistics and Probability

Then what.....ideas about what you would like to see us focus on in the future. Email ideas to jennywilcox@katm.org

Operations and Algebraic Thinking to Expressions and Equations/Functions to Algebra/Functions

Operations and Algebraic Thinking (K-5)

Kindergarten: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from; represent addition and subtraction; decompose numbers less than 10; fluently add and subtract within 5

1st grade: Represent and solve problems involving addition and subtraction; understand and apply properties of operations and the relationship between addition and subtraction; add and subtract within 20. Work with addition and subtraction equations. Including understanding the meaning of the equals sign

2nd grade: Represent and solve problems involving addition and subtraction.; add and subtract within 20; Work with equal groups of objects to gain foundations for multiplication.

3rd grade: Work with equal groups of objects to gain foundations for multiplication; Understand properties of multiplication and the relationship between multiplication and division; Multiply and divide within 100 (know multiplication facts by end of 3rd grade); Solve problems involving the four operations, and identify and explain patterns in arithmetic.

4th grade: Use the four operations with whole numbers to solve problems; Gain familiarity with factors and multiples; Generate and analyze patterns.

5th grade Write and interpret numerical expressions (including expressions with parentheses); Analyze patterns and relationships.

Expressions and Equations (6-8)

6th grade: Apply and extend previous understandings of arithmetic to algebraic expressions (including exponents on order of operations and writing equivalent expressions); Reason about and solve one-variable equations and inequalities; Represent and analyze quantitative relationships between dependent and independent variables.

7th grade: Use properties of operations to generate equivalent expressions; Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

8th grade: Work with radicals and integer exponents; Understand the connections between proportional relationships, lines, and linear equations; Analyze and solve linear equations and pairs of simultaneous linear equations;

Functions and Algebra (8-High School)

8th grade: Define, evaluate, and compare functions; Use functions to model relationships between quantities.

High School: Interpret the structure of expressions; Write expressions in equivalent forms to solve problems; Perform arithmetic operations on polynomials; Understand the relationship between zeros and factors of polynomials; Use polynomial identities to solve problems; Rewrite rational expressions; Create equations that describe numbers or relationships; Understand solving equations as a process of reasoning and explain the reasoning; Solve equations and inequalities in one variable; Solve systems of equations; Represent and solve equations and inequalities graphically; Understand the concept of a function and use function notation; Interpret functions that arise in applications in terms of the context; Analyze functions using different representations; Build a function that models a relationship between two quantities; Build new functions from existing functions; Construct and compare linear, quadratic, and exponential models and solve problems; Interpret expressions for functions in terms of the situation they model; Extend the domain of trigonometric functions using the unit circle; Model periodic phenomena with trigonometric functions; Prove and apply trigonometric identities.

Investigating Functions with a Ferris Wheel

Heather Lynn Johnson, Peter Hornbein, and Sumbal Azeem

What might students think when they hear the term function? A "machine" that takes in inputs and spits out outputs? Perhaps a symbol string, such as f(x) = x² + 1? A graph shaped like a U? Going further, what might students think when they hear that one quantity is a function of another quantity? For example, how might students interpret a statement such as "height is a function of distance"?

Teachers can use relationships between changing quantities to help students make sense of function. From this perspective, a function refers to a special kind of relationship between quantities. The phrase a function of describes a relationship between the quantities.

We provide a dynamic Ferris wheel computer activity that teachers can use as an instructional tool to help students investigate functions. We use a student's work to illustrate how students can use relationships between quantities to further their thinking about functions.

FERRIS WHEELS AND FUNCTIONS

Imagine the seats, or cars, of a turning Ferris wheel traveling along their circular path. Could you predict the height from the base of a car to the ground if you knew the distance the car had traveled within one revolution of the wheel? Could you predict the distance a car had traveled within one revolution of the wheel if you knew the height from the base of the car to the ground?

Teachers can ask questions like these to help students use relationships between quantities to investigate functions. In this Ferris wheel situation, students can determine a unique height for any given distance. In contrast, students cannot determine a unique distance for any given height.

Broadly, a function expresses a special kind of relationship between quantities. Chazan (2000) describes the
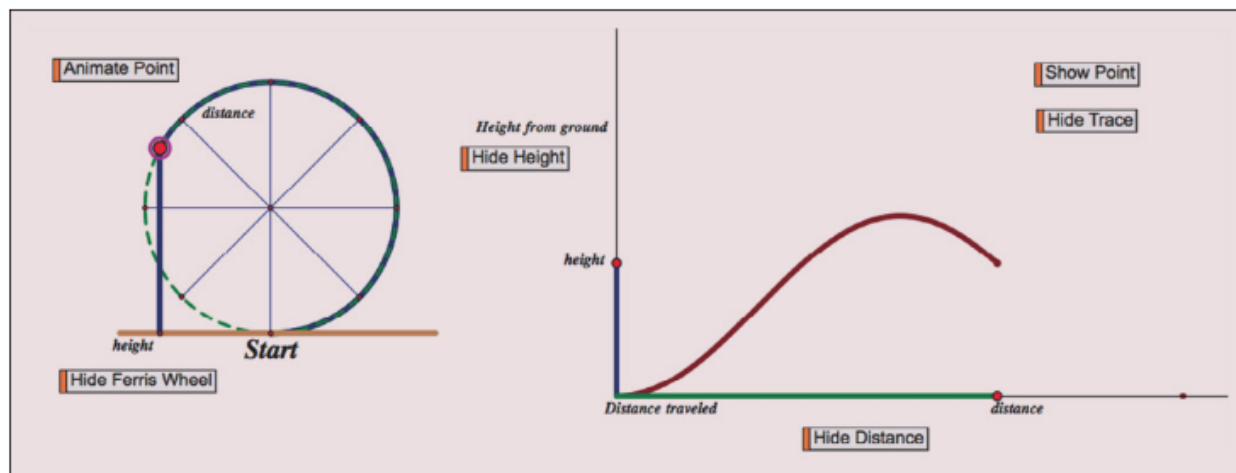

Fig. 1 A dynamic Ferris wheel computer activity relates height and distance.

For the Ferris wheel situation, we can use the term function to describe a special relationship between the quantities of height and distance. Specifically, height is a function of distance. Distance, however, is not a function of height. Put another way, height depends unambiguously on distance, but distance does not depend unambiguously on height.

By quantity, we mean more than just a label for a unit, such as inches or feet. We mean some "thing" that students can conceive of as being possible to measure (Thompson 1994a). For example, a student could conceive of the height from the ground to the base of a Ferris wheel car as something that she could measure by using a fixed distance between her thumb and forefinger.

USING AN INTERACTIVE COMPUTER ACTIVITY TO INVESTIGATE FUNCTIONS

Johnson used Geometer's Sketchpad® to design a dynamic Ferris wheel computer activity. (These files are available at www.nctm.org/mt as more4U content.) The activity links an animation of a turning Ferris wheel to dynamic graphs relating the quantities of height and distance (see fig. 1). When students press the Animate Point button, the car (represented by the red dot, fig. 1, left) moves in a counterclockwise direction around the Ferris wheel. As the car moves around the Ferris wheel, the linked graph changes dynamically.

The car moves at a constant rate, which would not happen on an actual Ferris wheel ride. To vary the rate at which the car moves, students can click and drag the car to control the motion. In addition, students can speed up or slow down the animation. By design, the dynamic graph represents only one revolution of the Ferris wheel so that students do not also have to keep track of the number of revolutions of the wheel.

When students work with graphs, it is useful for them to think about variation in individual quantities. Thompson (2002) recommends that a student use his finger as a tool to represent variation in individual quantities. Moving a finger horizontally or vertically, a student can track how individual quantities are changing with respect to passing time. In the dynamic Ferris wheel computer activity, students can manipulate and view the dynamic segments, shown on the horizontal and vertical axes (fig. 1, right) to represent how the height and distance will change.

Figure 1 shows all elements of the dynamic Ferris wheel computer activity (the Ferris wheel animation, the dynamic graph, and the dynamic segments). Teachers can hide or show different elements to vary students' opportunities for exploration.

Johnson also designed another version of the activity, which represents the height on the horizontal axis and the distance on the vertical axis (see fig. 2). By varying which quantities each axis represents, a teacher can provide additional opportunities for student exploration (see also Moore, Paoletti, and Musgrave 2013).

STUDENTS INTERACT WITH THE DYNAMIC FERRIS WHEEL

Johnson implemented the computer activity with a small group of ninth-grade students in an introductory algebra course. Most students began by reasoning about individual quantities of height and distance as changing with respect to passing time. We found that working with the dynamic Ferris wheel computer activity helped students form and interpret relationships between the changing quantities of height and distance.

We share the work of Ana, who interacted with the dynamic Ferris wheel computer activity in individual and small group sessions that were led by Johnson. Ana's work demonstrates the range of reasoning of all the students. We include Ana's work from four different sessions in which she investigated relationships between the changing quantities of distance and height.
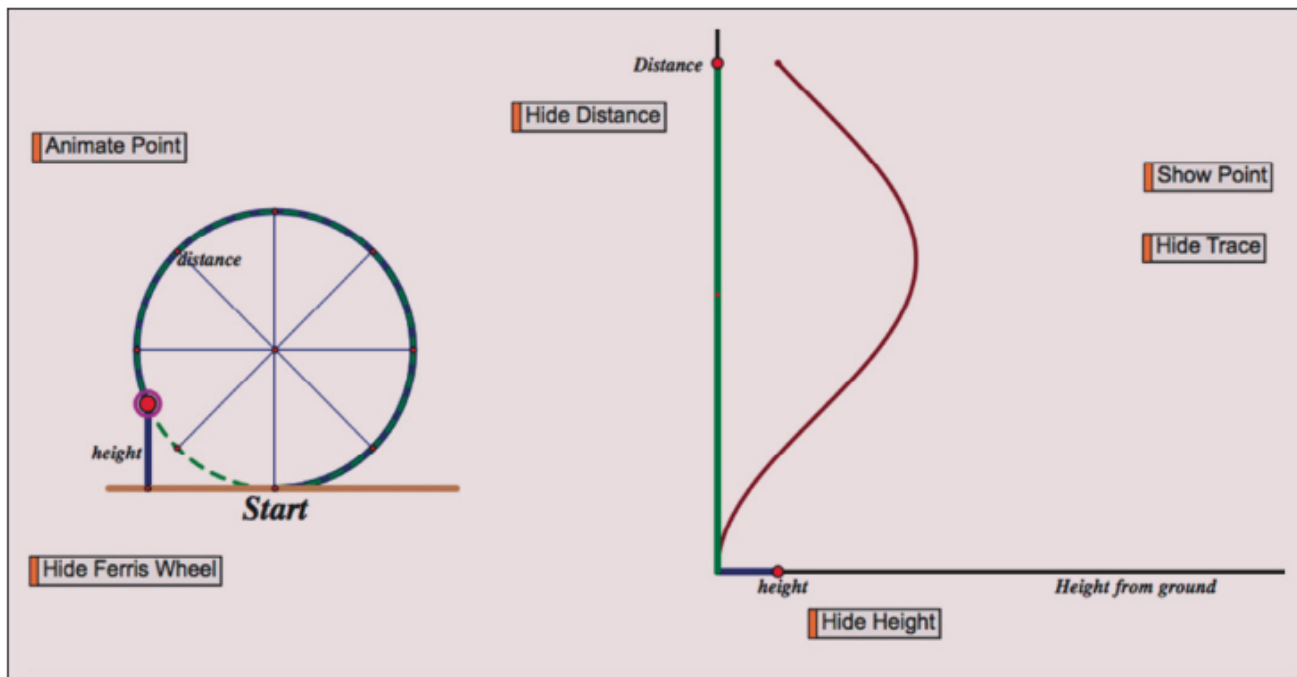
Fig. 2 A second dynamic Ferris wheel computer activity displays the height-distance relationship differently.
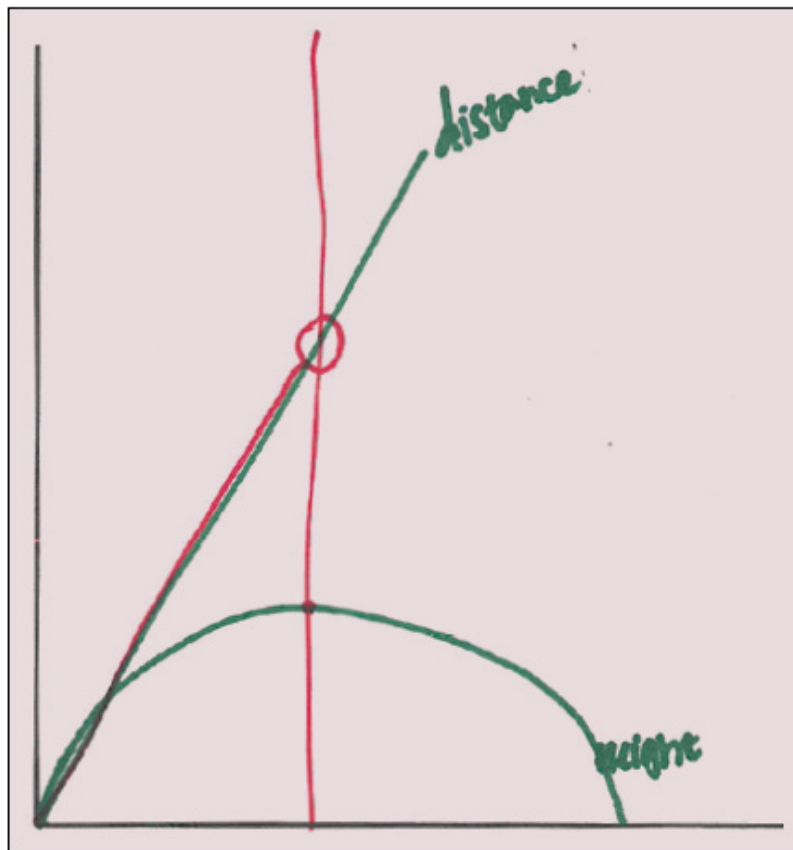

Fig. 3 In her first graph, Ana shows height and distance changing separately.

Session 1: Height and Distance Changing Separately

After students had seen only the Ferris wheel animation, Johnson asked them to sketch a graph relating a car's height from the ground and its distance traveled within one revolution of the Ferris wheel. Figure 3 shows Ana's graph.

Notice how Ana drew two graphs on the same pair of axes, one graph for distance and one graph for height. Also notice where Ana placed her labels. Rather than labeling the axes, she labeled each graph.

Ana drew a vertical line extending through both graphs to represent when the car was at the highest point on the Ferris wheel. Although she knew that to reach the highest point the car would travel half the distance around the Ferris wheel, her graph shows the car traveling far more than half the distance.

Next, students had opportunities to interact with the dynamic segments on the vertical and horizontal axes, representing height and distance, respectively. First, students predicted how each segment would change as the car moved around the Ferris wheel; then they used the dynamic graphs to confirm their predictions.

After making predictions, Ana viewed the dynamic graph shown in figure 1. Despite seeing a different graph, Ana did not make any changes to her graph (fig. 3). Ana was focusing on how the individual quantities of height and distance were changing with respect to passing time, which her graph represented.

Session 2: Height and Distance Changing Together

In this session, Ana described the changing height and distance in this way: "Distance is greater, greater, greater, and the height is greater, then it stops, and it goes back down." Johnson asked Ana to sketch a graph that represented the distance getting greater, and the height going up and then back down. Figure 4 shows Ana's new graph.

Notice how Ana's new graph included labels for the height and distance on the vertical and horizontal axes. In addition, she included written descriptions on the graph (e.g., "greater," "back down") to describe how the distance and height were changing.
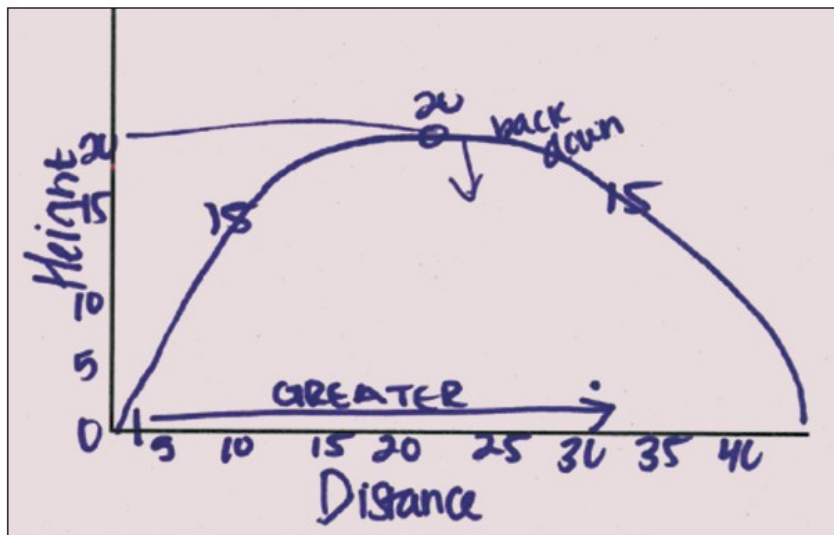

Fig. 4 Axis labels can give teachers insight to students' thinking.

Although Johnson did not ask Ana to do so, Ana included numbers on the axes before sketching her graph. She did not work from the numbers when sketching her graph, however. Rather, she began at the origin, then sketched the graph in one continuous motion, moving from left to right.

The shape of Ana's new graph (fig. 4) looks similar to that of the graph she drew for height, shown in figure 3. However, her new graph (in fig. 4) represents a relationship between height and distance. Her labels and descriptions show evidence of this.

Session 3: Height and Distance Changing Together

In this session, Johnson again asked Ana to sketch a graph representing the changing height and distance, before viewing a dynamic graph. This time the vertical axis represented distance and the horizontal axis represented height on Ana's graph.

First, Ana drew the highlighted part of the graph shown in figure 5. Rather than trying to reflect or rotate the shape of one graph to create a new graph, Ana used the changing distance and height. Ana drew arrows near the axes to represent the changing distance and height.

This time after Ana viewed the dynamic graph shown in figure 2, she noticed a new feature of the graph—the curvature. She said that how the graph "began" surprised her, and then she sketched the smaller, inner graph with different curvatures.

At this point, Ana was not sure why the dynamic graph curved the way that it did. However, she was beginning to develop a more nuanced understanding of the relationships between the changing height and distance.

Session 4: Height and Distance Changing Together

In this session, Johnson asked Ana to sketch a graph relating height and distance, with distance on the vertical axis and height on the horizontal axis. This time, the positive direction for height extended to the left. Figure 6 shows Ana's graph.

Ana's graph in figure 6 shows a relationship between the changing quantities of height and distance. Furthermore, it includes different curvatures to make distinctions between the ways in which the height and distance are changing together.

Before sketching this graph, Ana stated that she noticed that, at first, the car had begun to move around the Ferris wheel but that the height was still "about the same as

Fig. 5 In her third graph, Ana reversed the axes.

Fig. 6 Ana's fourth graph investigates curvature.

where you started." When Johnson asked Ana why her graph showed that, she focused on the lower right part of the graph (see arrow in fig. 6). Ana drew a vertical segment to represent how the distance was "still going." Next she drew a small horizontal segment (circled) to represent how the height was "still right here."

REPRESENTING CHANGING QUANTITIES

Ana's work illustrates three different ways in which students might use graphs to represent changing quantities, in this case distance and height. Table 1 makes distinctions between Ana's focus on quantities as changing separately or together, and highlights features of graphs she drew when representing the changing distance and height shown in the dynamic Ferris wheel computer activity.

Students often encounter graphs that represent two quantities (e.g., height and distance), neither of which is time. However, a student who is thinking about an individual quantity (or individual quantities) changing with respect to passing time may sketch a graph similar to Ana's first graph (fig. 3).
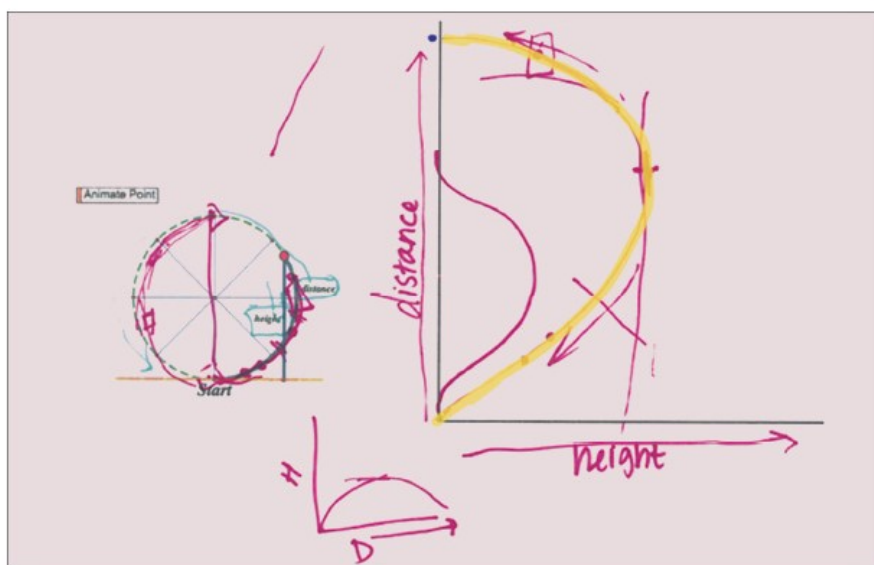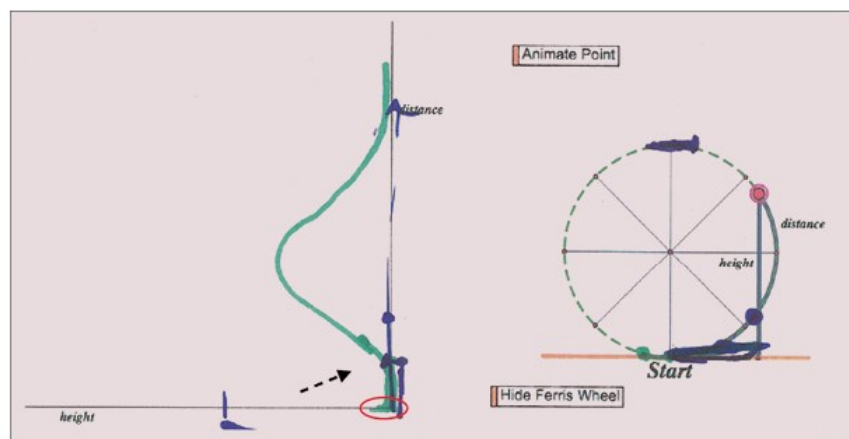
When students begin to use graphs to represent relationships between quantities, they may notice direction of change. For a Ferris wheel, for example, the height increases then decreases even as the distance continues to increase. A student who is thinking in this way may sketch a graph similar to Ana's second graph (fig. 4).

As students develop proficiency in using graphs to represent relationships between quantities, it is useful for them to focus on change occurring in an interval in which one quantity is increasing (or decreasing). For example, for the Ferris wheel, consider the interval in which the distance increases from zero to half the total distance. In this interval, the height increases slowly at first, then more quickly, then more slowly as the car reaches the maximum height. A student who is thinking in this way may sketch a graph similar to Ana's fourth graph (fig. 6).

VARIATION!

When students are studying function, it is important for them to think about quantities changing together. By interacting with the dynamic segments in the Ferris wheel computer activity, students have opportunities to explore how the individual quantities, height and distance, are changing with respect to passing time.

Once students demonstrate evidence that they are thinking about quantities changing together (e.g., by sketching graphs such as Ana's second graph), teachers can vary the representation of the quantities. For example, we reversed the axes on which we represented height and distance to help focus on changes in an interval in which one quantity is increasing or decreasing.

Although Ferris wheel problems are often used to introduce students to trigonometric functions, we recommend using this context much sooner, for students just beginning to study function. Students can benefit from opportunities to explore situations involving varying rates of change in conjunction with or prior to exploring linear relationships (e.g., Stroup 2002). Because the Ferris wheel activity incorporated varying rates of change, students had the opportunity to investigate change occurring in an interval in which one quantity was increasing (or decreasing).

A covariation perspective is not the only perspective that students should use when studying function. A correspondence perspective is also important, because students should understand that for a function, each input value has a unique output value. In fact, Ellis (2011) found that students who used relationships between quantities to investigate functions could move flexibly between covariation and correspondence perspectives.

Forming and interpreting relationships between changing quantities can provide a foundation for students' understanding of functions. When students have opportunities to think about quantities changing together, they can begin to use a covariation perspective on functions. The dynamic Ferris wheel computer activity is an example of one tool that teachers can use to foster students' thinking about quantities changing together.

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HEATHER LYNN JOHNSON, heather .johnson@ucdenver.edu, is an associate professor of mathematics education at the University of Colorado Denver. She is interested in middle and high school students' quantitative and covariational reasoning. PETER HORNBEIN, peter .hornbein@ucdenver.edu, recently retired from teaching high school mathematics and is now a graduate student in mathematics education at the University of Colorado Denver. His research interests lie in covariational reasoning and gesture. SUMBAL AZEEM, sumbal.azeem@ucdenver.edu, is a graduate student at the University of Colorado Denver. She teaches mathematics at Metropolitan State University of Denver.

KATM January Meeting Minutes

*Discussion held about membership and making sure there is a grace period between when a person's membership expires and the membership is actually ended.

*Scholarship report: No applicants for either scholarship at this point. It was discussed that we could award more than one scholarship for smaller amounts if people apply for less than $1000.

*We discussed what nominations we have for offices and who we might contact for run for available offices.

*We discussed changing the format of our Bulletin next year to be more of a business Bulletin, We would continue to offer articles and other resources to teachers on a section of the website. The Bulletin Editor and Webmaster will begin looking at how to accomplish this over the last half of the year.

*The Conference Advisory committee gave an update about the Topeka conference. The location is Seaman Middle School and the keynote speaker is Andrew Stadel. A preliminary schedule has been built.   Conference proposals can be submitted on the website. Discussion began for a site for 2018. We will look into Liberal, Maize or Manhattan.

*The Board had a discussion about the new proposed K-12 Math standards. We discussed how we could help our members know about and understand the changes being proposed.

*Zone Coordinators met to discuss how to get in touch with zone members.

Finding What Fits

Stephanie A. Casey

Statistical association between two variables is one of the fundamental statistical ideas in school curricula (Burrill and Biehler 2011; Garfield and Ben-Zvi 2004). Indeed, reasoning about statistical association has been deemed one of the most important cognitive activities that humans perform (McKenzie and Middlesen 2007). Students are typically introduced to statistical association through the study of the line of best fi t because it is a natural extension of their study of linear equations in mathematics. This is predominantly true for students in the United States; for example the authors of the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) ask that students in eighth grade learn about linear equations, linear functions, and the line of best fi t. A learning trajectory for linear regression study (Bargagliotti et al. 2012) begins with students finding and studying an informal line of best fi t, which refers to the idea that students are fitting a line, by eye, to data displayed in a scatterplot, without making calculations or using technology to place the line. Hence, it is found informally. For example, CCSSM states that students should know the following:

> Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fi t a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (p. 56).

The Common Core Standards Writing Team (2011) specified that this standard includes an expectation that students determine that the informal line of best fi t for data that has no association should be a horizontal line, and that a horizontal fi ted line implies that there is no association between the variables.

This article shares responses to a series of six tasks from a study analyzing students' understanding of the informal line of best fit. Thirty-three eighth-grade students in the United States were interviewed before they received instruction on the line of best fit (Casey 2015). Teachers can benefit from learning about this study in multiple ways. They can acquire meaningful tasks to implement with students when teaching informal line of best fit; gain knowledge of conceptions that students have about the line of best fit to plan for and manage instruction on the topic; and learn other implications for teaching the topic that resulted from the study.

DESCRIPTION OF TASKS

The first five tasks asked students to place a piece of piano wire to represent the line of best fi t for data presented in a scatterplot and justify why they placed it there. Piano wire was chosen for its rigidity and thinness, although in other settings the tasks have been completed equally well using raw spaghetti or pipe cleaners. The fi ve tasks implemented are displayed in figures 1 and 2. The data were chosen on purpose. The plots (1) presented data from real-world contexts that were familiar to students; (2) had eight points, which was a manageable number; and (3) did not contain outliers or influential points. They progressed from plots displaying a strong positive association (tasks 1 and 2), to plots displaying a relatively strong negative association (tasks 3 and 4), to a plot displaying no association (task 5).

Fig. 1 Scatterplots were presented to students (a), who then placed best-fit lines (b). The least-squares regression line is in red.
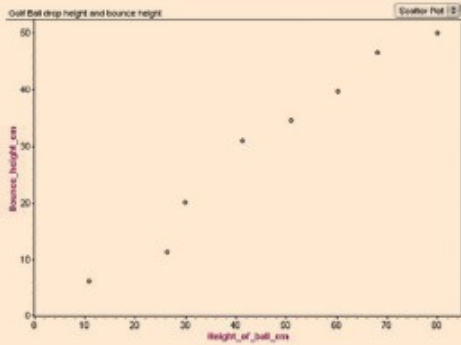
(a) Task 1
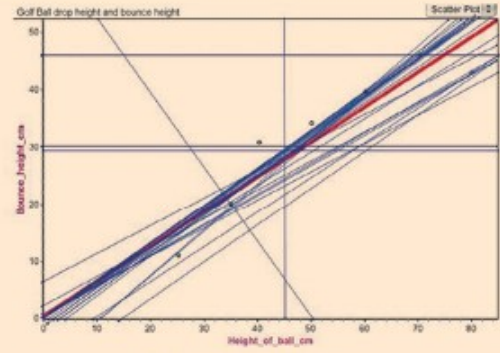
(b) Students' lines

(a) Task 2

(b) Students' lines

(a) Task 3

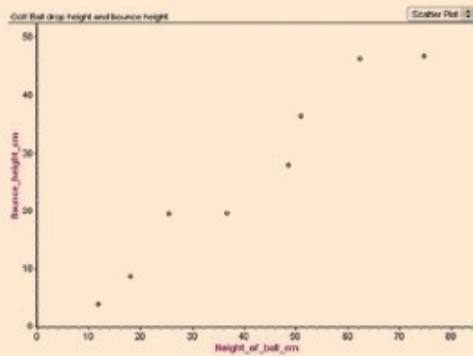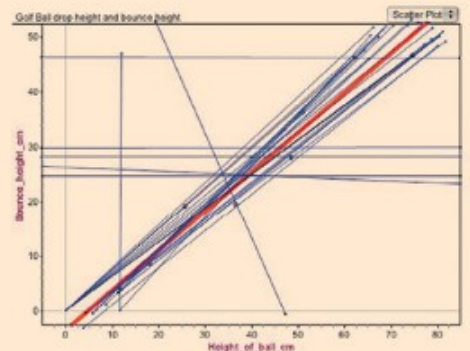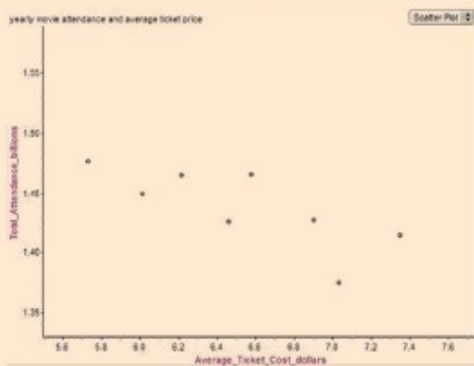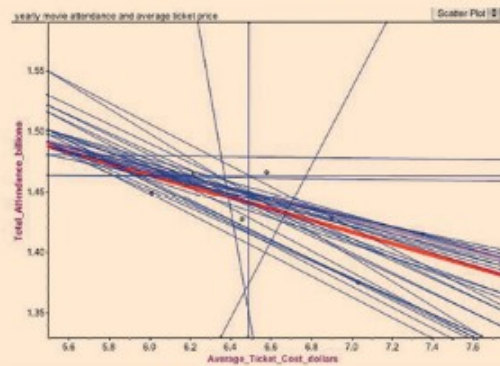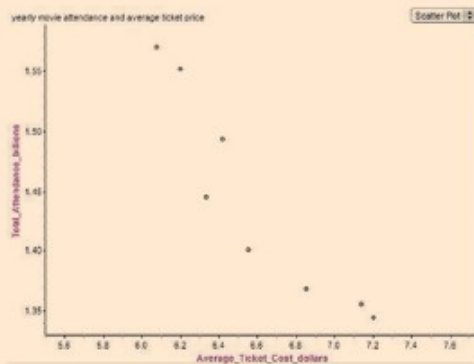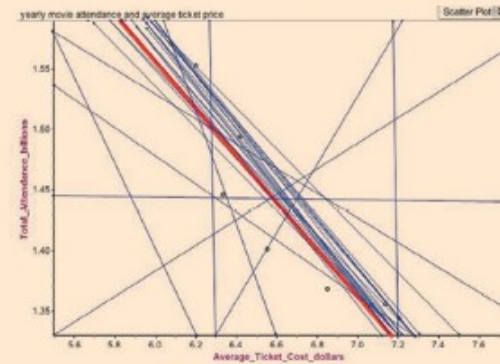(b) Students' lines

(a) Task 4

(b) Students' lines

RESULTS: THE LINE OF BEST FIT WITH LINEARLY ASSOCIATED DATA

The first notable result was that a sizeable number of students (9), when asked to find the line of best fit on the first task, wanted to bend the wire to connect the points on the scatterplot. For instance, Marcus (a pseudonym, as are all student names) asked, "Wouldn't it be like the line that starts here [the origin] and like, connects . . . connects all these points, right?" Some students struggled to conceive of the line of best fi t as a line that did not necessarily go through all the points, likely because this differed from graphs of linear functions

that these students had been studying in mathematics. When statements like this occurred after students were presented with the first task, the interviewer redirected by explaining that the goal was to find the line of best fit. Because lines are straight, students were not to bend the wire. After receiving this instruction, all the students were able to complete the tasks, suggesting that this same redirection may be effective in a classroom setting.

Figure 1 presents all 33 students' lines for tasks 1 through 4. The least squares regression line plotted in red provides a visual image of the accuracy of the placed lines for these tasks. These displays show that there was considerable variability in the placed lines' locations. The majority of the lines were reasonably accurate in that they were generally close to the least squares regression line, but a substantial number of lines were placed inaccurately. Looking at the criteria that students used for placing the lines provided more insight into the process (see table 1 for the students' criteria and the number of different students who used each criterion).

Table 1 reveals that the criteria that students naturally devised for finding the informal line of best fit were numerous

Criteria	Number of Students
Through as many points as possible	13
Equal number of points on both sides	8
As close to all the points as possible	7
Reflect the relationship the variables have on the basis of context knowledge	5
Halfway between the lowest and highest points	3
Through the first and last points	3
Starting from the first point then maximizing the number of points it goes through	2

Table 1 The description and number of students who used various criteria showed contrasting views.

and varied in their viewing of the data set as a whole.   Some criteria used the selection of specific points (e.g., lowest and highest, first and last) to determine the line, ignoring the rest of the data set. Other criteria, such as "equal number of points on both sides" and "as close to all the points as possible," showed that the students were considering the data in their entirety when finding the line of best fit. The third most commonly used criterion, "as close to all the points as possible," is the one encouraged by CCSSM (CCSSI 2010) and is in agreement with the approach of the least-square regression line.

A closer examination of the criteria for and the location of lines placed on task 2 provided greater insight regarding students' conceptions of the line of best fi t. Figure 1, task 2 (b) shows the informal best-fi t lines that students placed on task 2 along with the least-squares regression line. The thirteen criteria identified by the 33 students when placing the line on this task (see fi g. 3) resulted in a large number of lines placed near the least-squares regression line. However, most generally ran parallel to or split the least-squares regression line, with very few following it. This occurred because of the predominance of the most points and equal number criteria and the decision of students employing those criteria to force their line to go through one of the last two points.

Fig. 2 Task 5 explored elementary students' height and shoe size.
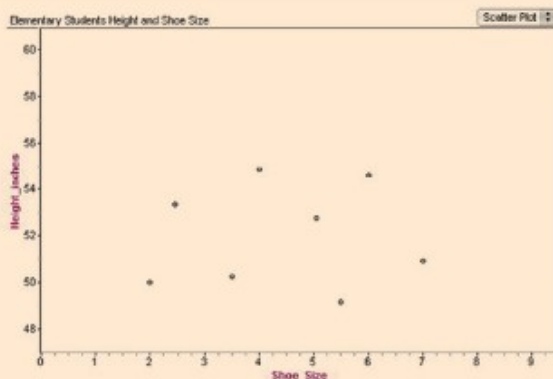
(a) Task 5
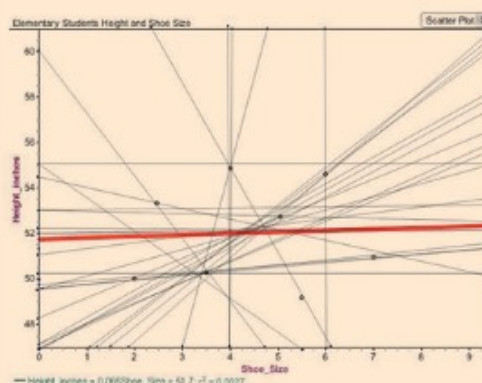
(b) Students' lines

Equal number points each side (7)
Closest to all the points (6)
Most points (4)
Through first point and last point (4)
Context (4)
Horizontal through highest point
Horizontal halfway between highest and lowest points
Horizontal at low value
Through middle point
Through maximum point
Through bottom-left corner and direction
Connect point on y-axis of highest point's y-coordinate
to point on x-axis of x-coordinate of lowest point

(c) Students' approaches

A closer examination of the lines placed by students so that an equal number of points would be on each side of the line (see fi g. 4) revealed that this criterion resulted in remarkably different lines. Three of these lines were relatively accurate, with one following the least-squares regression line nearly exactly. However, the other two lines were inaccurate because they were placed horizontally. These students' explanations about the horizontal placement sound appropriate ("I'm putting it in the middle"; "It's at the average"), and a teacher would be inclined to think that these students understood the topic. However, these students applied "middle" and "average" in an univariate rather than bivariate sense and therefore placed their lines at the "middle" or "average" of the bounce height only.

These explanations and actions should raise cautions for teachers when teaching the topic: avoid solely teaching students to place the line so that an equal number of points are on each side and probe what your students mean by "middle" and "average" in a bivariate data analysis setting.

RESULTS: THE LINE OF BEST FIT FOR DATA WITHOUT ASSOCIATION

The presentation of task 5's scatter plot that displayed no association evoked different responses and approaches from the students than the previous four tasks (see fig. 2). The time it took students to complete this task was considerably longer than the other tasks, and many students studied the plot in silence for a substantial time (around twenty seconds) before responding. Six students initially commented that they did not see a general trend or

direction in the plot and were confused about what to do. One student, however, commented that she did not see a general trend in the plot but correctly used that observation to place the line both horizontally and halfway between the lowest and highest points because "it's not decreasing or increasing." This is the conclusion we wanted to help all students make (Common Core Standards Writing Team 2011), but it was evidently not a natural conclusion for students.

There were various locations for the placed lines on this task compared with the previous four tasks. Figure 2b displays all the lines placed by the students (Sasha said, "I have no idea," and did not place a line), along with the least-squares regression line. The criteria employed by students on this task ordered by frequency of use are listed in figure 2c . The number of students choosing a criterion was shown in parentheses if used by multiple students.

It is notable that relatively few students placed lines close to the least-squares regression line. Even those students who claimed to place the line closest to all the points, as the least-squares regression essentially did, were unable to do so accurately on this task. Another important observation to make from figure 2 is that a large number of the placed lines have positive slopes likely because students expected that bigger shoe sizes correlated to bigger heights. Therefore, they placed their lines with positive slopes to show that relationship although it was not exhibited in the data in the plot. One teaching implication is that students should be asked to work with data sets such as this one that disagree with assumed relationships to encourage students to discuss what to base the placement of the line of best   fit on: contextual knowledge, the data at hand, or some combination of the two.

Fig. 3 Students used several criteria when placing the line for task 2.

Most points
Through (0, 0) and last point
Through (0, 0) and most points
Through first point and last point

Equal number points each side
Through a point and equal number
    points each side

Closest to all the points
Through (0, 0) and closest to all
    the points
Through first point and last point
Horizontal through highest point
Horizontal at average of y-variable
Through middle point, intersect
    middle of x-axis
Vertical to hit x-axis at value of
    lowest point, upper end across
    from highest point

EVALUATING LINES OF BEST FIT

A classroom of students informally fitting a line of best fit to data will result in numerous lines, so it is important that students consider how to evaluate lines to determine which line best fits the data. To this end, a sixth task was presented to students in the study. The scenario for this task was that two students, Angelo and Barbara, were asked to complete task 1 but had different solutions (see fig. 5). Students were asked, "Which student's line fits the data better and why?" The task was designed so that Angelo and Barbara's line placement would be similar; however, Angelo's line (A) goes through two points, whereas Barbara's line (B) was closest to all the points (it was the least squares regression line) but did not go through any points.

One-third (11) of the students in the study chose line A; the other two-thirds (22) chose line B. Seven of the 11 students who chose line A stated that they preferred it because it went through some of the points, including 3 students whose dominant criterion for placing lines was through the most points. Thus, teachers can anticipate that a sizeable number of their students will likely need learning experiences to change their conception that it is more important to go through, rather than be near, all points (the criteria included in CCSSM 8.SP.A.2; CCSSI 2010).

Nineteen students who chose line B explained that it was closer to all the points than line A. One notable result was that 7 of the 10 students whose dominant criterion for placing the line of best fit on tasks 1–5 was "through the most points" chose line B as the better line, shifting to note that being closest to all the points was most important for the line of best fit. For 3 of these students, their progression through the tasks involved a transition away from the criteria of "through the most points" that they had used for the earlier tasks.

As Sasha described, she "started out thinking like Angelo but now sees that Barbara's is better." For others, completing this task was an illuminating experience. It allowed them to evaluate whether going through or being near all the points was more important. For a number of students, that evaluation process helped them see why being closer to all the points created a better line of best fit. Teachers are encouraged to use this task for those same purposes in their classrooms.

Fig. 4 Students placed lines with an equal number of points on each side.
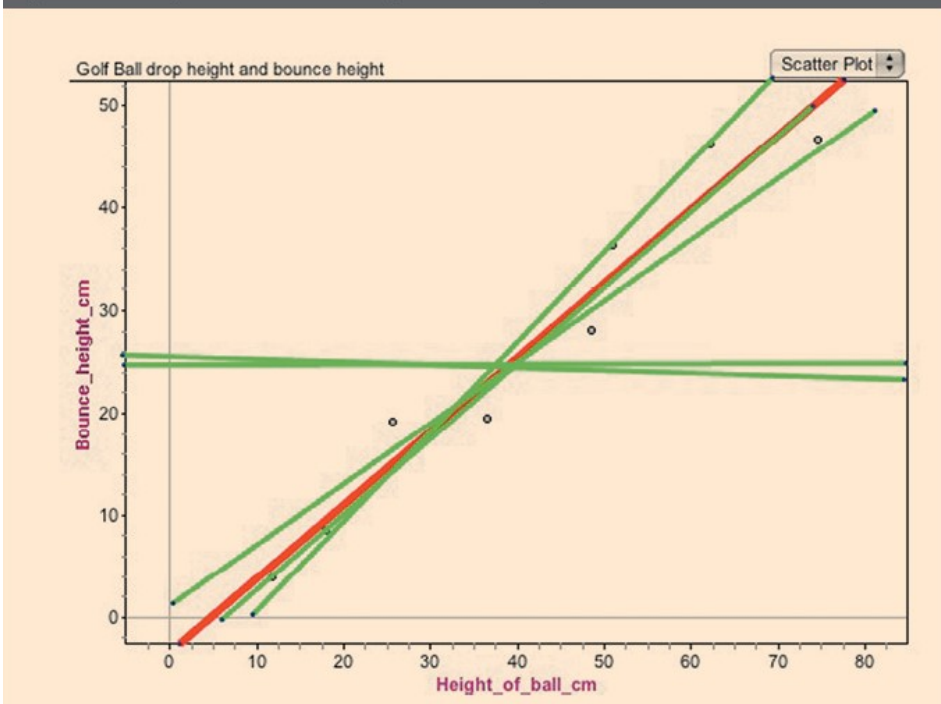
MEANINGFUL IDEAS AND ESSENTIAL KNOWLEDGE

The informal line of best fi t is a relatively new addition to the mathematics curriculum with the implementation of CCSSM (CCSSI 2010); however, it is extremely important because it serves as the foundational topic for the study of the fundamental concept of statistical association. The tasks and student responses to them described how students conceive of the informal line of best fit. In so doing, instruction might be crafted to meet students' learning needs.

Fig. 5 Task 6 from the interview asked students, "Which line fits the data better and why?"
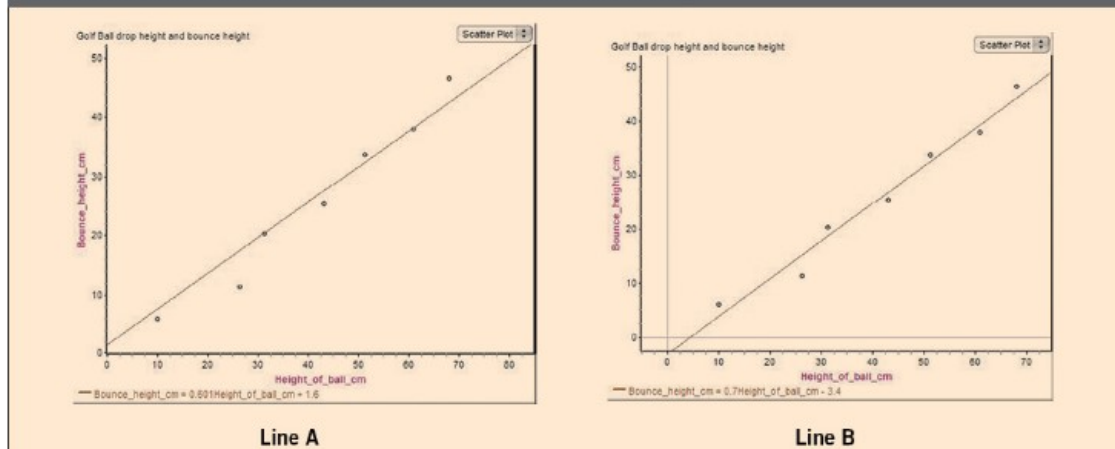
Line A   **Line B**

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Just Say Yes to Early Algebra

Ana Stephens, Maria Blanton, Eric Knuth, Isil Isler, Angela Murphy Gardiner

Mathematics educators have argued for some time that elementary school students are capable of engaging in algebraic thinking and should be provided with rich opportunities to do so. Recent initiatives like the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) have taken up this call by reiterating the place of early algebra in children's mathematics education, beginning in kindergarten. Some might argue that early algebra instruction represents a significant shift away from arithmetic-focused content that has typically been taught in the elementary grades. To that extent, it is fair to ask, "Does early algebra matter?" That is, will teaching children to think algebraically in the elementary grades have an impact on their algebra understanding in ways that will potentially make them more mathematically successful in middle school and beyond?

Plenty of evidence certainly exists that elementary school students can think algebraically about particular concepts. For example, we know that students can develop a relational understanding of the equal sign (Carpenter, Franke, and Levi 2003; Falkner, Levi, and Carpenter 1999); generalize important arithmetic relationships such as the Commutative Property of Multiplication (Bastable and Schifter 2008; Schifter 1999); and use representations such as tables, graphs, and variable notation to describe functional relationships (Blanton 2008; Carraher et al. 2006). However, it is also important to know how children think algebraically across a comprehensive set of algebraic concepts in content domains that, at first glance, might not seem deeply connected.

In this article, we share findings from a research project whose goal is to study the impact of a comprehensive early algebra curricular experience on elementary school students' algebraic thinking within a range of domains including generalized arithmetic, equivalence relations, functional thinking, variables, and proportional reasoning. We focus here on the performance of third-grade students who participated in our early algebra intervention on a written assessment administered before and after instruction. We also discuss the strategies these students used to solve particular tasks and provide examples of the classroom activities and instructional strategies that we think supported the growth we saw in students' algebraic thinking.

We believe the research presented here paints a compelling picture regarding the potential for elementary school students to successfully engage with a range of early algebraic concepts, and we believe that sharing this with educators—who are increasingly expected to develop children's algebraic reasoning (CCSSI 2010)—is important.

Our early algebra intervention

Two third-grade classrooms with a combined total of thirty-nine students participated in our intervention. Students' regular mathematics curriculum contained little algebra. Our instructional sequence consisted of approximately twenty one-hour early algebra lessons throughout the school year that took the place of students' regularly scheduled mathematics instruction for that day. Each lesson began with small-group discussions of previously taught concepts, and then new concepts were introduced through small-group problem solving and whole-class discussion. One member of our research team, a former elementary school teacher, taught all the lessons.

In this article, we discuss students' responses to a representative sample of items from the pre-assessment and post-assessment (see Blanton et al. 2015 for a more thorough presentation of assessment results) and the nature of the instruction that supported their learning.

Results

How do you think your own students would respond to a representative sample of assessment items (see fig. 1)? Students who participated in our instruction made significant gains in their abilities to view the equal sign as a relational symbol, identify arithmetic properties (e.g., the Commutative Property of Addition), write variable expressions to represent unknown quantities, and generalize and express functional relationships.

In addition to whether students responded correctly to each assessment item, we were also interested in the types of strategies they used and whether the strategies that students used at the end of our instruction reflected more algebraic ways of thinking than those they had used before our instruction. We found that students who had the opportunity to engage in early algebraic thinking throughout the course of the school year tended to approach the assessment items more algebraically and were more apt to "look for and make use of structure," one of the Common Core's (CCSSI 2010) Standards for Mathematical Practice (SMP 7, http://www .corestandards.org/Math/Practice/). In what follows, we discuss the strategies that students used to solve the items (see fig. 1) and highlight the structural thinking that we observed.

How did students "look for and make use of structure"?

Equality

The fact that many students view the equal sign as an operational symbol meaning "give the answer" has been well documented (e.g., Behr, Erlwanger, and Nichols 1980; Carpenter, Franke, and Levi 2003). We likewise found that the vast majority of students were unsuccessful with the equality items during pretesting (see fig. 1a) and gave responses indicating they viewed the equal sign operationally by placing a 10 or 14 in the blank in $7 + 3 = $ ___ $+ 4$ or by stating that $57 + 22 = 58 + 21$ is false because, for example, "$57 + 22 = 79$, not 58." However, students clearly came to view the equal sign as a relational symbol over the course of our instructional intervention (see fig. 1a). For many of these students, growing knowledge of the equal sign as meaning "the same value as" in arithmetic and algebraic equations led them to compute sums on both sides of these equations to find the missing value in $7 + 3 = $ ___ $+ 4$ or to determine the validity of $57 + 22 = 58 + 21$. However, many of them went a step further and developed the ability to view these equations structurally and successfully solve these items without using computation. By posttest, 16 percent of students gave an explanation indicating they solved $7 + 3 = $ ___ $+ 4$ by attending to structure (e.g., "Four is one more than three, so the blank must be one less than seven"), and 29 percent of students gave an explanation indicating they solved $57 + 22 = 58 + 21$ by attending to structure (e.g., "Fifty-eight is one more than fifty-seven, and twenty-two is one more than twenty-one, so it's true").

Generalized arithmetic

One of the core areas of early algebra is generalized arithmetic, whereby students deepen their arithmetic understanding by noticing and representing regularity and structure in their operations on numbers. When asked whether $39 + 121 = 121 + 39$ was true or false, none of the students who responded correctly during the pretest gave an explanation that relied on the equation's underlying structure. They tended, rather, to compute the sums separately on each side of the equal sign and find $160 = 160$. At the posttest (see fig. 1b), however, 66 percent of students provided this type of explanation (e.g., "True, because $121 + 39$ is just $39 + 121$ in reverse").
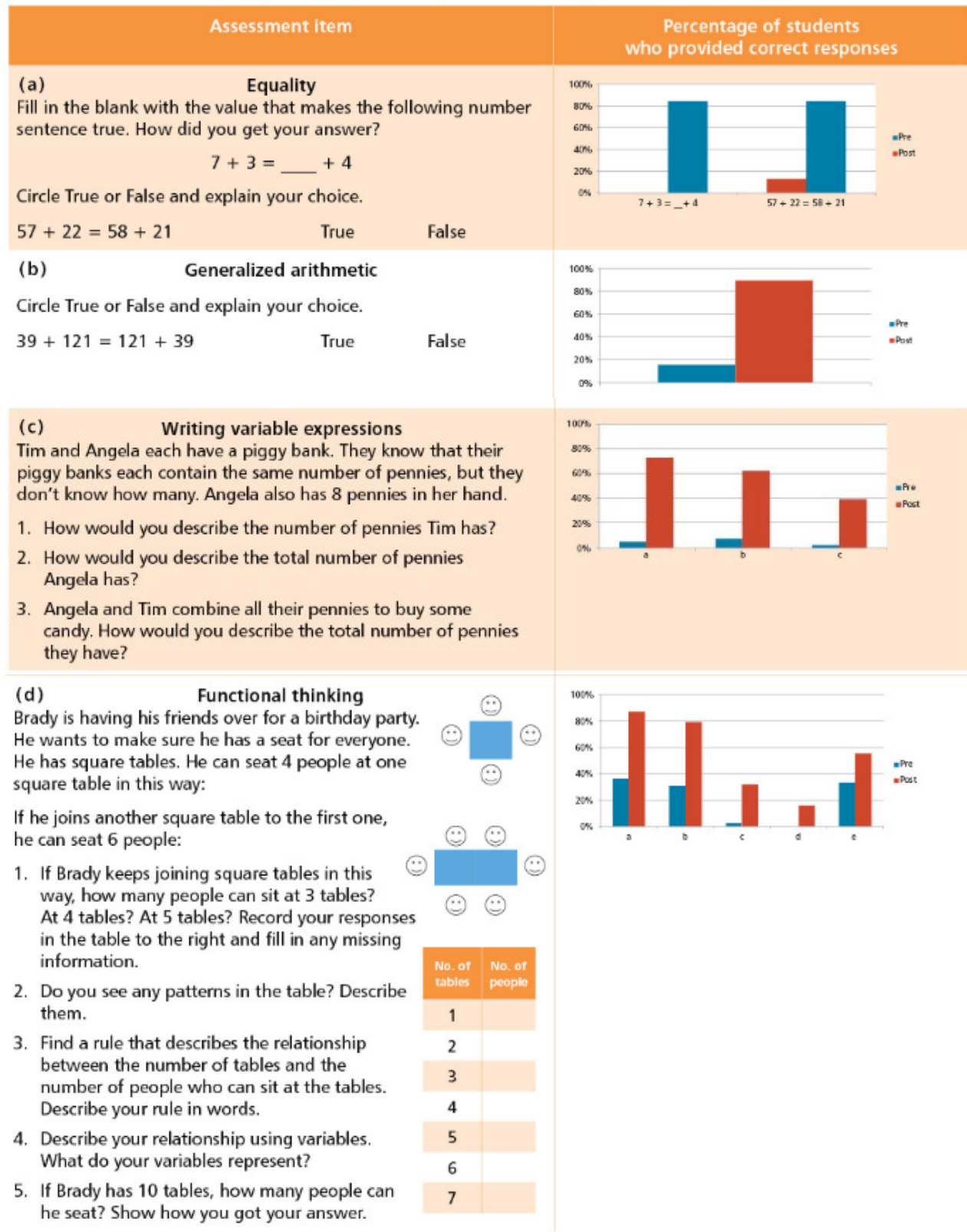
Writing variable expressions

Students who confront an unknown quantity are often uncomfortable with this ambiguity and want to assign a specific value rather than use a variable (Carraher, Schliemann, and Schwartz 2008). Likewise, we found that students were unable to represent unknown quantities symbolically at pretest time (see fig. 1c) and that those who

Researchers found that with instruction, students made significant gains in their abilities to view the equal sign as a relational symbol, identify arithmetic properties, write variable expressions to represent unknown quantities, identify recursive patterns, and generalize and express functional relationships in both words and variables.

Student performance on a representative sample of assessment items

Assessment Item	Percentage of students who provided correct responses
(a) **Equality** Fill in the blank with the value that makes the following number sentence true. How did you get your answer? $7 + 3 = $ ____ $+ 4$ Circle True or False and explain your choice. $57 + 22 = 58 + 21$ True False	
(b) **Generalized arithmetic** Circle True or False and explain your choice. $39 + 121 = 121 + 39$ True False	
(c) **Writing variable expressions** Tim and Angela each have a piggy bank. They know that their piggy banks each contain the same number of pennies, but they don't know how many. Angela also has 8 pennies in her hand. 1. How would you describe the number of pennies Tim has? 2. How would you describe the total number of pennies Angela has? 3. Angela and Tim combine all their pennies to buy some candy. How would you describe the total number of pennies they have?	
(d) **Functional thinking** Brady is having his friends over for a birthday party. He wants to make sure he has a seat for everyone. He has square tables. He can seat 4 people at one square table in this way: If he joins another square table to the first one, he can seat 6 people: 1. If Brady keeps joining square tables in this way, how many people can sit at 3 tables? At 4 tables? At 5 tables? Record your responses in the table to the right and fill in any missing information. 2. Do you see any patterns in the table? Describe them. 3. Find a rule that describes the relationship between the number of tables and the number of people who can sit at the tables. Describe your rule in words. 4. Describe your relationship using variables. What do your variables represent? 5. If Brady has 10 tables, how many people can he seat? Show how you got your answer.	

No. of tables	No. of people
1	
2	
3	
4	
5	
6	
7	

responded to this item did so by choosing a numerical value to represent Tim's number of pennies (e.g., "Tim has ten pennies"), even though the item specifically stated that the quantity is unknown.

It is often assumed that young students are not "developmentally ready" to work with variables and should instead work exclusively with concrete representations. Our findings suggest, however, that students who are provided with the appropriate experiences can engage quite successfully with symbolic representations. In response to question 1 in figure 1c, no student assigned a specific numerical value to the unknown quantity at posttest and, in fact, 74 percent used a variable to represent the quantity (e.g., "Tim has n pennies").

Further, students' posttest responses to questions 2 and 3 highlight their abilities to attend to mathematical structure and treat expressions as single objects. We found that 63 percent of students were able to use variable notation to represent Angela's number of pennies in a way that connected to their representation of Tim's number of pennies in question 1. In other words, these students understood that if n represented Tim's number of pennies, then Angela's number of pennies could be best represented by n + 8. Similarly, 39 percent of students provided a representation in part c that related to those in questions 1 and 2. For example, if students represented Tim's number of pennies as n in question 1, then these students might represent the combined number of pennies for Tim and Angela as n + n + 8 in question 3. We believe this indicates that these students were using variables with understanding and were thinking structurally by building on previously established expressions.

Functional thinking

Functional thinking involves reasoning about and expressing how two quantities vary in relation to each other (Blanton 2008). This algebraic domain unfortunately often receives little attention in the elementary grades (Blanton and Kaput 2011) even though it is a significant part of CCSSM in later grades. We found, however, that with instruction, young students can learn to recognize and express functional relationships. As figure 1d shows, students made gains in their abilities to complete function tables, identify recursive patterns, generalize functional relationships, and represent these generalizations in both words and variables. See Isler and her colleagues' (2015) detailed account of student performance on this assessment item and the classroom activities that contributed to the development of students' functional thinking.

How did students' algebraic thinking develop?

How did students—during the course of one school year—develop such sophisticated ways of thinking about a wide range of algebraic concepts? While focusing on the algebraic domains mentioned above, students were also asked to engage in four algebraic thinking practices that are central to the discipline and align to a great extent with the Common Core's SMP (CCSSI 2010). In what follows, we discuss each of these practices and use students' work exploring even and odd numbers as examples to illustrate what this thinking looked like in our classrooms and what it might look like in yours.

First, students were routinely posed tasks that encouraged them to generalize mathematical relationships and structure. This type of thinking occurs when students notice relationships or structure in arithmetic operations, expressions, equations, or function data that can be generalized beyond the given cases. For example, students in our classrooms were asked to explore representing numbers with cubes so that they might come to identify two types of numbers—even and odd (see fig. 2).

Students noted that the numbers in the table alternated between having zero and one "leftover." They concluded that even numbers—the ones with zero leftover cubes—always have a "buddy." They also noticed that even numbers, when divided into two rows of cubes, form a rectangle; whereas odd numbers always have one cube sticking out by itself.

Students explored even and odd numbers by representing numbers with cubes.

How many pairs?
Use your cubes to complete the following table for the given numbers.

No.	No. of pairs created	No. of leftover cubes
3		
4		
5		
6		
7		

What do you notice? What kinds of numbers have no cubes left after all pairs are made? What kinds of numbers have a cube left? Write a sentence to describe each of your observations.

FIGURE 3

Students explored sums of even and odd numbers. The researchers discovered that manipulatives are useful tools in promoting the identification of relationships and mathematical structure.

1. Jesse is adding two even numbers. Do you think his answer will be an even number or an odd number?

2. Jesse is adding two odd numbers. Do you think his answer will be an even number or an odd number?

3. Jesse is adding an even number and an odd number. Do you think his answer will be even or odd?

After exploring properties of even and odd numbers, students were asked to think about sums of even and odd numbers by working on the task shown in figure 3. As students explored both representing sums of numbers with cubes and computing specific sums of evens and odds, they began to notice important structures in even and odd numbers and their sums. We found that in the context of generalized arithmetic in particular, manipulatives are useful tools that help promote the identification of relationships and mathematical structure.

Once students had recognized mathematical relationships, we often asked them to represent generalizations. Students can use various notational systems—words, symbols, tables, graphs, and pictures—to represent their generalizations. In the case of the questions we posed (see fig. 3), our students used words to express conjectures, such as "An even number plus an even number is an even number" and "An even number plus an odd number is an odd number." In a few years, these students should be able to use symbolic notation to express an even number as $2n$ and an odd number as $2m + 1$ (for any integers n and m). Natural language, however, can be a useful scaffold for developing an understanding of symbolic notation.

Students in our intervention were also asked to justify generalizations. When asked to justify a generalization they have expressed verbally or symbolically, students often begin by offering numerical examples. We found this to be true in the case of students' explorations with even and odd numbers, with students saying, for example, "I know an even plus an even is an even because $2 + 4 = 6$." It is important, however, that students learn to appreciate the limitations of "justification by example" and move toward making general arguments. Think about how our teacher encouraged this shift in students' thinking by considering the following excerpt of classroom dialogue:

Student 1: We could say that when you add an even number plus an even number, the sum will be even.

always work? Have we shown or tried enough examples to be sure that this will always work?

Student 1: No, we should probably try a few more. [Students add more even numbers and write sums.]

Teacher: So, how are we feeling? Do you still feel that an even plus an even will always be even?

Student 2: Yes, because I tried a bunch of examples and it works for all of them.

Teacher: Great. I agree. I think that when we add an even plus an even, the sum will always be even. But, why? Why does this always work?

[Students give more examples.]

Teacher: Yes, I agree. You have shown me a great number of examples, but why? What did we learn about even numbers when we were exploring a little while ago?

Student 3: Even numbers always have pairs!

Teacher: OK, so could that help us answer why an even plus an even is an even?

Student 3: Yes, because when we add even numbers, we don't ever start with any leftovers, everyone has a pair; so we can add them together, and everyone will always have a pair.

Notice that asking such questions as "Do you think this will always work?" and "Why does this always work?"—as well as referring students back to their previous "definitions" of even and odd numbers—helped students move beyond examples-based reasoning.

Talking about numbers in general can be difficult for children when they are accustomed to working with specific values. Sometimes, however, specific examples can be used in such a way that students' justifications do not depend on the specific numbers used. Consider, for example, how the following student used cubes to justify that the sum of two odd numbers is an even number:

I did it with blocks. So, I took 9 blocks, and I added it to 11. If you look at the blocks alone, 9 and 11, they each have a leftover, but when you put them together, their leftovers get paired up, so you have an even number. [See fig. 4.]

Notice that although this student's justification used nine blocks and eleven blocks, there is nothing special about these specific numbers. Any odd numbers could have been chosen to make the argument. Furthermore, the student did not need to calculate in the process of justifying the generalization. This type of justification is sometimes referred to as "representation-based" reasoning (Russell, Schifter, and Bastable 2011) because it relies on the use of a physical or visual

FIGURE 4

This student used representation-based reasoning to justify why the sum of two odd numbers is an even number.

representation as a bridge to a general argument. A good strategy is to question students about the specific examples they choose—"Did you have to count those cubes?" or "Does it only work for your example?"—to encourage them to engage in representation-based reasoning and begin to appreciate the power of general arguments.
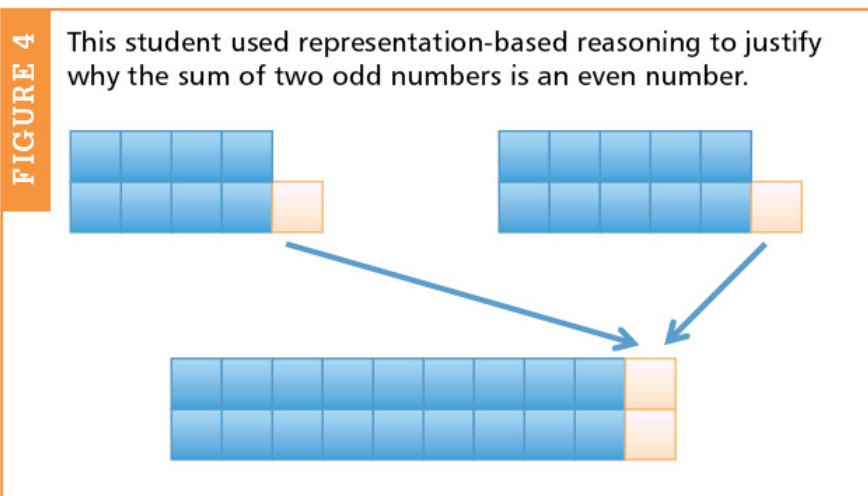
Finally, students in our classroom were often encouraged to reason with generalizations. This occurs when students make use of generalizations to solve problems. Students often do this naturally, without being asked to do so and without explicitly thinking about the generalizations they are using. For example, when asked whether the sum of three odd numbers would be even or odd, our students were often able to build on the already-established generalization that the sum of two odd numbers is an even number. One student explained, for example, that two odd numbers equal an even number and—if you have an even number, it is all paired up. If you add that to an odd number, which has a leftover, you can never get rid of the leftover. It still has nothing to pair with, so your answer will always be odd.

Part of engaging students in thinking algebraically involves posing tasks that encourage the use of a particular generalization and then helping students make the taken-for-granted generalization explicit.

Can young students be successful in algebra?

Overall, our study's results reveal that elementary school students who experience a comprehensive and sustained early algebra education—that is, across multiple algebraic domains and spanning an entire school year—can successfully engage with a variety of algebraic content that is often reserved until middle school or later. The ability to think structurally is an important aspect of algebraic thinking (Kieran 2007), and we found that third-grade students in our study were capable of this type of reasoning. Keeping these results in mind, we encourage you to work with your students in the algebraic domains discussed here and engage them in the important algebraic thinking practices of generalizing mathematical relationships and structure, expressing generalizations, justifying generalizations, and reasoning with generalizations.

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About the Authors

Ana Stephens, acstephens@wisc.edu, is an associate researcher at the Wisconsin Center for Education Research at the University of Wisconsin– Madison. She is interested in the development of students' and teachers' algebraic reasoning. Maria Blanton, maria_blanton@terc.edu, is a senior scientist at TERC in Cambridge, Massachusetts. Her interests are in the development of children's algebraic thinking and in understanding its impact on their success in the formal study of algebra. Eric Knuth, knuth@education.wisc.edu, is a mathematics education professor at the University of Wisconsin–Madison. His work focuses on the development of students' mathematical reasoning. Isil Isler, isler@wisc.edu, is a graduate student at the University of Wisconsin–Madison. She is interested in algebraic thinking and reasoning and proof in the elementary and middle grades. Angela Murphy Gardiner, angela_gardiner@terc .edu, is a senior research associate at TERC. A former elementary school educator, she brings her classroom experience and knowledge to her research projects, where she enjoys working with students as they explore early algebra.

This month, our Vice President Elementary selected the following article for inclusion in our Bulletin.

To Teach Math, Study Reading Instruction

From more than 50 years of teaching experience, I've learned that elementary school teachers are typically more comfortable teaching reading. They delight in watching students become readers. They enjoy students' rapt attention when they read books aloud to them. They love discussing the ideas that books inspire. They have a variety of teaching strategies and ways to organize students for reading instruction.

I've also learned, sadly, that when it comes to math, the same qualities don't always exist. Teachers often tell me that they come to math with a combination of trepidation, fear of the subject matter, and a general "uncomfortableness." A sense of confidence is often missing from teaching math, as are feelings of joy and delight.

Math time is often serious and tense. Rigor and seriousness are essential, but so are the excitement and creativity that teachers generate when teaching language arts.

Of course, in some important ways, learning to read and learning math are different. One disparity became clear to me while reading a novel on an airplane. Several times I came upon words that were unfamiliar. Still, I kept on reading and didn't get lost. Even if I had been home, I doubt that I would have checked a dictionary. Most likely, I would have continued reading and still made sense of what was going on. But in math, if you miss one blip in class, you fall off the learning ladder. You have to back up and start again to pick up the piece you missed. That's the way math is—you can't skip a step and stay with it—whereas in reading, it's possible to continue and let the context of what you're reading carry you.

Similarities do exist between learning to read and learning math. For example, both involve skills. But this similarity also leads to another difference: For reading, there's one gatekeeper skill, decoding. While decoding alone isn't sufficient for reading proficiency, it's the essential skill that gives readers access to the entire world of printed matter. Unfortunately, this isn't the case for math. As early learners, children learn to count and add and subtract small numbers; next, they learn about place value and working with greater numbers; then they move on to multiply and divide. They do all of this first with whole numbers, and then face fractions and decimals and how to add, subtract, multiply, and divide them as well. In math, there's no one gatekeeper skill that students can practice and perfect: The concepts and skills build and build. It can be daunting.

But other similarities exist. For example, when a child is learning to read, everybody knows that reading proficiency is all about bringing meaning to the printed page. I can "read" anything in Spanish, since I've studied some Spanish, yet still not understand much of what I'm reading. I can "read" about string theory, yet not fully grasp the concepts. No one finds it acceptable to think of a child as a proficient reader if he or she can pronounce the words but doesn't understand the material. Comprehension is the key to being a successful reader.

The same standard should hold true for math. If children have memorized the math facts and can perform computational procedures, teachers often think of them as proficient. But we've seen over and over again how children can borrow, carry, bring down, or invert and multiply without understanding why the procedures work or how to apply them to problem-solving situations. The standard for math should be the same as the standard for reading: bringing meaning to the printed symbols. In both situations, skills and understanding must go hand in hand. The challenge is: How do we help students develop meaning and make sense of what they do?

A few years ago, when I was leading a professional-learning session at a school, I asked the teachers to list what they thought was important in their instructional program for reading and language arts. I had them do this first in small groups, and then we went around the room and I recorded as each group reported something from list. Here are some examples of what they said:

- *We want our students to love reading.*
- *We want them to develop good word-attack skills.*
- *We want them to read fluently.*
- *We use a variety of teaching strategies—shared reading, guided reading, independent reading, read-alouds.*
- *We include comprehension from the very start.*
- *We ask children to make predictions about what might come next in a story.*
- *We do a lot of making inferences.*
- *We want students to decipher meaning from contexts.*
- *We ask them to pose questions about what they're reading.*

We want them to identify what's important and what's not as important in what they read.

After we had gone around the room several times, and I had filled two sheets of chart paper with their ideas, the teachers were still excited. They had a lot to say, and they radiated enthusiasm.

I then said, "Let's review each statement, change the reference from reading to math, and see what we discover." Doing this made some of the teachers uncomfortable. Some admitted that the focus on comprehension and thinking skills that was so prevalent in their language arts instruction was missing from their math instruction. Others noticed that the confidence they felt in articulating what they did during reading instruction didn't exist when describing their math instruction.

We had an interesting conversation about the relationship between fluency in reading and fluency in math. While comprehension was key to reading fluency, with math they often felt relieved when students could compute accurately. One teacher commented, "Sometimes I know that students don't understand why they are borrowing or carrying, but I don't know what to do." This experience was a start for us to discuss essential questions about math instruction: What are your beliefs about teaching math? What are your goals for the students?

A major challenge at the elementary level is teachers' content knowledge. Most teachers don't have the same content knowledge in mathematics that they have for reading and language arts. I'm convinced that you can't teach what you don't understand, and I think that learning more about the math they have to teach is an important area for teachers' professional learning.

But, just as I know that children learn best when they connect new learning onto their existing knowledge and skills, the same holds true for teachers. How can we connect literacy and math, so that teachers bring the strengths they have with language arts instruction to their math teaching? How can teachers make links between mathematics and language arts pedagogy that will enable them to engage children with math in the same way they bring children to the wonder of reading?

One way is for teachers to think about leading classroom discussions in mathematics as they often do when teaching language arts. Probing students' thinking during math lessons is valuable, so that the goal is not only getting correct answers, but also explaining why answers make sense.

Teachers typically ask students to explain when they've given an incorrect answer. "Are you sure about that?" is often a signal to students that their answer is wrong. But it's important even when their answers are correct to ask: "Why do you think that?" "How did you figure that out?" "Who has a different idea?" "How would you explain your answer to someone who disagreed?" It's useful to have students comment on their classmates' answers as well, asking them to explain what a peer said in their own words, or asking students if they have a different way to explain the answer. If students are stuck, it's sometimes useful to have them turn and discuss the problem with a partner and then return to a whole-class discussion.

Instructional practices like these support the development of skills and help students cement and extend their understanding.

There's much for us to think about to help teachers teach math more effectively. But I think we can make headway if we take the two most important areas of the curriculum—reading and math—and look at them side by side to analyze what's the same, what's different, and what we can learn from one to enhance the other.

About the Author: Marilyn Burns is a teacher, provider of in-service training and professional development, conference speaker, and writer. The author of more than a dozen books for children, she has created more than 20 professional-development resources for teachers and administrators, including Math Solutions. Her widely read book About Teaching Mathematics was released in its fourth edition last month.

KATM Elections are Coming!

Election information will be coming on March 1. Be watching for it, and don't forget to vote!

Are you interested in trying your hand at presenting at a conference….but you're a little nervous???

NEKATM is here to help!

NEKATM (Northeast KATM) is hosting an event at AJ's Pizzeria at 5:00 on March 30. We will go over the basics of presentation planning. We' d love to see you there for some pizza and planning!

KLFA

Kansas Learning First Alliance January Meeting Addresses the New Year

Representatives from Kansas Learning First Alliance member organizations met to gain information on new accreditation protocol and tax reform initiatives during a recent meeting held January 5, 2017 in Topeka, KS.

Dr. Bill Bagshaw, KSDE Assistant Director of Teacher Licensure & Accreditation shared information on the Kansas Educational Systems Accreditation (KESA) model, more specifically the Outside Visitation Teams (OVT) that will be an integral part of the accreditation process in the future. He explained the training manual for chairs of these teams is being created to deliver strong preparation for team leaders. Different from the earlier accreditation process, KESA will accredit systems (usually districts) rather than individual buildings. An integral piece of the process is the active engagement that building and district level teams of educational professionals and community members will have in the process. He also provided information on the five-year cycle roll-out. For more information please see KESA Information.

Heidi Holliday, Executive Director of Kansas Center for Economic Growth, and Haley Pollock, Director of Communication and Outreach for Kansas Action for Children, provided information about the tax reform initiative developed by the Rise Up Kansas coalition and which provides a solution to the current and growing crisis for education programs and resources, especially for young children in Kansas if the governor's vision remains in place. They clearly explained why comprehensive tax reform is critical and simply addressing a piece (such as closing the LLC loophole) will not bring about the changes needed. The dire results of securitizing any amount of the Master Settlement Agreement (MSA) fund (aka tobacco settlement fund) , causing a dismantling of the Children's Initiative Fund, was discussed. For more information see Rise Up Kansas.

The Kansas Educational Leadership Institute (KELI) was welcomed as a new member organization. Dr. Rick Doll, Executive Director of KELI shared with meeting participants the KELI's mission to provide excellent professional learning and resources for educational leaders across Kansas. KELI is a collaborative body formed by Kansas School Superintendents Association, United School Administrators, Kansas Association of School Boards, Kansas State Department of Education and Kansas State University.

Mark Desetti, Leah Fliter, and Tom Krebs provided a legislative update highlighting the changes from the recent election. Committee chairs appear to be more moderate, and several of the new legislators have experience in the work of schools. It was noted there is a significant increase of new legislators in both House and Senate who ran on the platform of being supportive of public schools. The critically important work on the new school finance formula will be starting with the new session.

The next (and final) meeting for KLFA for the 2016-2017 academic year will be held on April 11, 2017 at the KNEA Building (715 SW 10th Ave. Topeks, KS)For more information about KLFA, visit the KLFA Website and/or look for "Kansas Learning First Alliance" on Facebook.

NCTM Update:

By Stacey Bell, Past President NCTM

My name is Stacey Bell and I am pleased to be the NCTM Rep for KATM. As I stated in our last Bulletin, NCTM has a new website design and has been focusing on developing its Affiliate Site for its members. As an affiliate of NCTM, KATM is able to now post our upcoming events on this new site for neighboring states to see. And likewise, we are able to see what other affiliates are doing around us. You should check it out at http:// www.nctm.org/affiliates/

In other news, NCTM has published a new book, *Principles to Actions: Ensuring Mathematical Success for All*. Below is NCTM's description of the book found at http://www.nctm.org/Store/Products/Principles-to-Actions--Ensuring-Mathematical-Success-for-All/
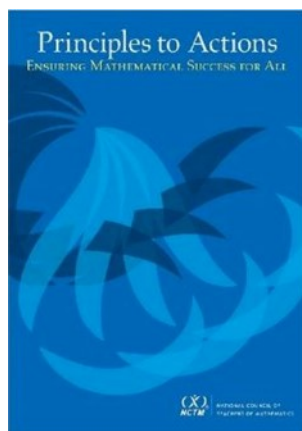
The widespread adoption of college- and career-readiness standards, including the Common Core State Standards for Mathematics, presents a historic opportunity to improve mathematics education.

What will it take to turn this opportunity into reality in every classroom, school, and district?

Continuing its tradition of mathematics education leadership, NCTM has defined and described the principles and actions, including specific teaching practices, that are essential for a high-quality mathematics education for all students.

Principles to Actions: Ensuring Mathematical Success for All offers guidance to teachers, specialists, coaches, administrators, policymakers, and parents:

- Builds on the Principles articulated in Principles and Standards for School Mathematics to present six updated Guiding Principles for School Mathematics

- Supports the first Guiding Principle, Teaching and Learning, with eight essential, research-based Mathematics Teaching Practices

- Details the five remaining Principles—the Essential Elements that support Teaching and Learning as embodied in the Mathematics Teaching Practices

Identifies obstacles and unproductive and productive beliefs that all stakeholders must recognize, as well as the teacher and student actions that characterize effective teaching and learning aligned with the Mathematics Teaching Practices

With *Principles to Actions*, NCTM takes the next step in shaping the development of high-quality standards throughout the United States, Canada, and worldwide.

Call for Nominations!!!

The KATM Board is currently taking nominations to fill the following positions in the upcoming Board election. We are looking for educators that are interested in taking a leadership role in the field of Math Education throughout the State of Kansas. You can nominate yourself or someone that you know that has demonstrated a passion for advancing math in our state as well as someone that has a lot to offer in the way of supporting teachers. Please email Fred Hollingshead, Past President (hollingsheadf@usd450.net), with nominations and contact info of the nominee or fill out the online nomination form found at katm.org. Regular members in good standing are eligible for positions on the KATM Board. Nominations need to be completed by February. Elections will be held online at www.katm.org in March.   A notice will be sent to remind you to vote.

Positions available for the upcoming election:

President-elect * 4-year term

The president-elect will serve for one year before then becoming president for a year, and then past-president for two years. The president-elect will assume the duties of president when needed. As president, the elected individual will preside over all KATM events and business meetings. The president will conduct the business of KATM as directed by the Executive Board and will represent KATM at a variety of functions, meetings, and conferences. The president is responsible for the overall functioning of the organization with assistance from the officers and Board members. As the past-president taking office in even-numbered years, this position will serve as the community relations representative for 2 years. This person shall be responsible for assuring communication between the Association and legislative, executive, and administrative branches of the government of Kansas.

Vice President – College * 2-year term

The vice president for college will attend all meetings and conferences and will assist the president in conducting the business of KATM. Each vice president will also encourage membership and will promote issues of special interest to their level represented in addition to serving on various committees as assigned. The person elected to this position will act as a liaison to college teachers.

Vice President – Middle School * 2-year term

The vice president for middle school will attend all meetings and conferences and will assist the president in conducting the business of KATM. Each vice president will also encourage membership and will promote issues of special interest to their level represented in addition to serving on various committees as assigned. The person elected to this position will act as a liaison to middle school teachers.

Nominee Biographies

President Elect

Todd Flory is a 4th grade teacher at Wheatland Elementary School in Andover, Kansas. In addition to serving on his building and district's leadership teams, Todd is a Skype Master Teacher, Microsoft Innovative Educator Expert, Buncee Ambassador, Sway Champion, Microsoft Certified Educator, Google Certified Educator, and the 2016 PBS Digital Innovator Kansas lead. As an educator, Todd focuses on providing global collaboration and real-life, passion-based learning experiences for his students. He has spoken on these topics at state and national education conferences, including at ISTE and FETC. Todd believes that teachers need to create global citizens in a global classroom to empower students to shape their future and the world's.

Vice President College

Carrie L. La Voy, Ph.D—Nominee for VP College

Carrie La Voy, Ph.D., is running for KATM's VP of College. Dr. La Voy, a multi-term lecturer in the department of Curriculum and Teaching, joined the KU School of Education faculty full-time in the fall of 2010. Previously she worked as an adjunct faculty member at the University of Kansas, Johnson County Community College, Ottawa University, and Haskell Indian Nations University. These positons gave her the opportunity to teach both mathematics courses and education courses. Her professional background also includes teaching 8th grade mathematics, elementary gifted education, and pre-school education.

At the University of Kansas, Dr. La Voy teaches mathematics methods courses for pre-service teachers at the elementary and secondary level. She also teaches graduate course in the department of Curriculum and Teaching. She is a member of KATM, NCTM, and AMTE. She currently serves as the faculty advisor for the student chapter of KU-KNEA.

Dr. La Voy's research interests include methods of differentiating assessment and instruction, investigating why students struggle with mathematics, and improved methods of training pre-service teachers in mathematics education. She has received grants to support service learning and teacher mentoring programs at some of the high schools where her students complete field work. She has given many presentations related to mathematics education, including speaking at the annual AMTE conference and at NCTM conferences.

Dr. La Voy currently serves on the professional advisory board at Horizon Academy, a private, fully accredited school specializing in serving children with learning disabilities. Recently, she was named a KU Diversity Scholar. The Diversity Scholars Program brings together a small group of faculty engaged in discussion and collaboration around incorporating greater attention to diversity and more inclusive practices in their college classes.

Tanya Smith—-Nominee for VP College

My name is Tanya M. Smith and I bring you greetings from Kansas City Kansas Community College where I am the Mathematics Department Coordinator/Associate Professor. I am a native of Kansas City, Missouri graduating from Lincoln College Preparatory Academy. My educational background consist of a Mathematics degree from the University of Arkansas at Pine Bluff, MBA from Baker University, math graduate hours from the University of Missouri-Kansas City, and currently working on my dissertation at Capella University.

Tanya Smith Bio continued—I have taught in higher education for the past 16 years; however, I have been working in education for the past 20 years. My education experience started in Duncanville, Texas where I taught 7th and 8th grade math. I have had the opportunity to do some Adjunct teaching at several colleges and universities that allowed me to experience different educational environments. In 2005, I started a tutoring business, T2 Tutorial Service, which provided math and reading tutoring to K-12th students as well as college students. My services also provided in house math tutoring to some schools in the Kansas City Missouri School District as well as professional development, college readiness, and financial aid workshops to parents and students wanting to attend college. For the past two summers, I have participated in the Summer Bridge program at University of Missouri-Kansas City as the Math Enrichment Instructor for incoming freshman.

I have served on several committees at KCKCC: Faculty Senate, Faculty Development, Coordinators Committee, General Education Review, Program Review, Developmental Education, Academic Policies, Institution Strategic Planning, Math Redesign, Mentoring & Student Support, and Academic Calendar Committee. I am very active in my community participating in community service projects, STEM workshops and panels, and college fairs in the Kansas City Metro area. I am excited about the opportunity for the position of Vice President – College for the Kansas Association of Teachers of Mathematics. I truly believe in building a better relationship between K-12th and colleges and universities. Starting college can be overwhelming emotionally and financially. Improving the relationship between school districts and colleges can help ease some financial burdens by providing more dual credit courses giving high school students the opportunity to take college credit-bearing courses taught by college-approved high school teachers. Building this relationship will also start an early college awareness to elementary and middle school students. My hope is that with this position, Kansas City Kansas Community College can be a strong representative for KATM in the Kansas City metro area. I am married and a mother of 3 amazing young men.

Vice President Middle School

Blake Carlson, Topeka 501

Blake is currently serving as VP Middle School. He was appointed to fill the term when the prior VP Middle School moved. He works for Topeka 501.

Josh Lee, Valley Center Middle School

I want to be a leader for middle school math teachers in Kansas. As my building's department head, I have had the opportunity to lead a department of change agents in seeking the best ways to reach students. I'm excited about the possibility of doing this at the state level with other teachers who are as excited about math education. I have 13 years of experience working with students of all levels, gaining the kind of well rounded experience that would be important in a state leader.

Working for ALL math teachers across Kansas.

KATM
Kansas Association of Teachers of Mathematics

An event for all math educators!

2017 KATM Annual Conference

OCTOBER 16, 2017
Seaman MIDDLE SCHOOL
TOPEKA, KS

ADD TO YOUR MATHEMATICS TOOL BELT

Number Sense | Thinking | Problem Solving | and more!!

- Standards Update from KSDE
- Opportunities to network with other math educators (kindergarten to college) in the state
- Awesome Break-Out Sessions
- Zone Meetings with Zone Coordinators

Keynote Speaker: **ANDREW STADEL**
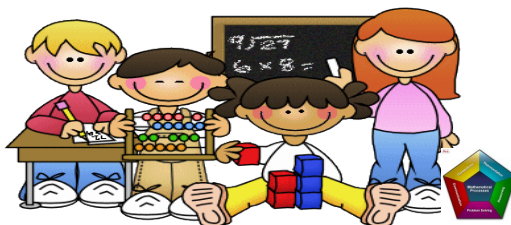He is well-known for his widely-acclaimed
ESTIMATION 180. Check him out at
www.estimation180.com.

Learn more at www.katm.org. Looking forward to celebrating and exploring mathematics together!!

KATM Cecile Beougher Scholarship
ONLY FOR ELEMENTARY TEACHERS!!

A scholarship in memory of Cecile Beougher will to be awarded to a practicing Kansas elementary (K-6) teacher for professional development in mathematics, mathematics education, and/or mathematics materials needed in the classroom. This could include attendance at a local, regional, national, state, or online conference/workshop; enrollment fees for course work, and/or math related classroom materials/supplies.

The value of the scholarship upon selection is up to $1000:

- To defray the costs of registration fees, substitute costs, tuition, books etc.,

- For reimbursement of purchase of mathematics materials/supplies for the classroom

An itemized request for funds is required. (for clarity)

REQUIREMENTS:

The successful candidate will meet the following criteria:

- Have a continuing contract for the next school year as a practicing Kansas elementary (K-6) teacher.

- Current member of KATM (if you are not a member, you may join by going to www.katm.org. The cost of a one-year membership is $15)

APPLICATION:

To be considered for this scholarship, the applicant needs to submit the following no later than June 1 of the current year:

1.   A letter from the applicant addressing the following: a reflection on how the conference, workshop, or course will help your teaching, being specific about the when and what of the session, and how you plan to promote mathematics in the future.

2.   Two letters of recommendation/support (one from an administrator and one from a colleague).

3.   A budget outline of how the scholarship money will be spent.

Notification of status of the scholarship will be made by July 15 of the current year. Please plan to attend the KATM annual conference to receive your scholarship. Also, please plan to participate in the conference.

SUBMIT MATERIALS TO:

Betsy Wiens
2201 SE 53rd Street
Topeka, Kansas 66609   ***Go to www.katm.org for more guidance on this scholarship***

Back to table of contents

Capitol Federal Mathematics Teaching Enhancement Scholarship

Capitol Federal Savings and the Kansas Association of Teachers of Mathematics (KATM) have established a scholarship to be awarded to a practicing Kansas (K-12) teacher for the best mathematics teaching enhancement proposal. The scholarship is for up to $1000 to be awarded at the annual KATM conference. The scholarship is competitive with the winning proposal determined by the Executive Council of KATM.

PROPOSAL GUIDELINES:

The winning proposal will be the best plan submitted involving the enhancement of mathematics teaching. Proposals may include, but are not limited to, continuing mathematics education, conference or workshop attendance, or any other improvement of mathematics teaching opportunity. The 1-2 page typed proposal should include

- A complete description of the mathematics teaching opportunity you plan to embark upon.
- An outline of how the funds will be used.

An explanation of how this opportunity will enhance your teaching of mathematics.

REQUIREMENTS:

The successful applicant will meet the following criteria:

- Have a continuing contract for the next school year in a Kansas school.
- Teach mathematics during the current year.

Be present to accept the award at the annual KATM Conference.
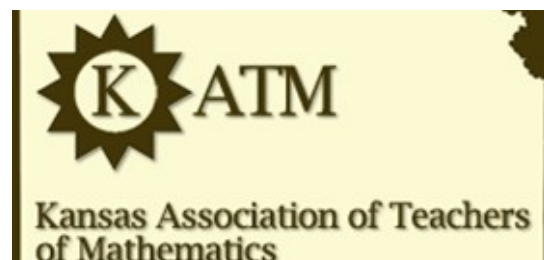
APPLICATION:

To be considered for this scholarship, the applicant needs to submit the following no later than **June 1 of the current year.**

- A 1-2 page proposal as described above.

Two letters of recommendation, one from an administrator and one from a teaching colleague.

PLEASE SUBMIT MATERIALS TO:

Betsy Wiens, Phone: (785) 862-9433, 2201 SE 53rd Street, Topeka, Kansas, 66609

KSDE Update

Upcoming Events
Kansas Excellence in Math and Science Education Conference
Save the date!
June 12th – 14th, 2017
Hutchinson, KS

Three full days of rich profession development in the areas of math and science.

Math Standards Review
Review process began in Spring 2016 and new standards are set to be sent to the State Board of Education for approval in June 2017. Detailed timeline, committee members and meeting dates can be found at http://community.ksde.org/Default.aspx?tabid=6151.

KSDE will be holding town hall type meetings in February and March details are listed below:

What is the KSDE proposing:

A complete copy of the standards can be accessed at: http://community.ksde.org/LinkClick.aspx?fileticket=f9XiZKWgQMw%3d&tabid=6151&mid=15110. A summary/highlight of the changes are:

- Some content moved to different grade levels.
- Document format is easier to read.
- Document is interactive.
- Teacher and student glossary.
Language clarifications.

Who is affected by these proposed standards:

Kansas teachers of mathematics in elementary, middle and high schools, as well as students and parents of both public and private accredited schools.

How to provide feedback:

Those who are interested should access the following link: http://bit.ly/Math-Standards-Feedback. This link provides people an opportunity to offer written feedback.

Additionally, oral and written comments may be offered at the following public meeting sites:

City	Time	Date	Location
Dodge City	6 p.m. – 7:30 p.m.	2/1/17	The Learning Center
			308 W. Frontview Rm 1
			Dodge City, KS 67801

Wichita 6 p.m. – 7:30 p.m.   2/6/17 Wichita Area Technical College
                               Main Campus - National Center of Aviation Training
                               4004 N Webb Rd,
                               Building 300, Room S211
                               Wichita, KS 67226

Topeka 6 p.m. – 7:30 p.m.   2/8/07 Highland Park High School
                               Media Center
                               2424 SE California Ave
                               Topeka, KS 66605

Pittsburg   6 p.m. – 7:30 p.m.   2/15/17   Pittsburg High School
                               Room 301 (the Little Theater)
                               1978 East 4th Street
                               Pittsburg, KS 66762

Salina   6 p.m. – 7:30 p.m.   3/1/17 Lakewood Middle School
                               1135 Lakewood Circle
                               Salina, KS 67401

Hays   6 p.m. – 7:30 p.m.   3/8/17 Rockwell Administration Center
                               Toepfer Board Room
                               323 W. 12th Street
                               Hays KS, 67601

Please share your feedback and encourages everyone to attend a meeting.
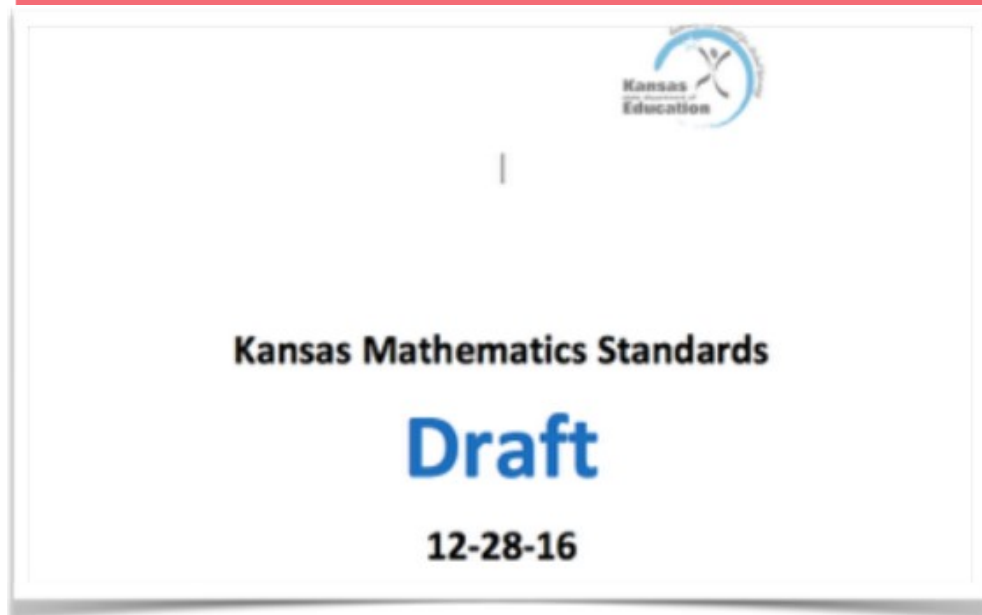
Training Opportunities
KSDE consultants and/or trained trainers can come to your district and provide training around many areas in mathematics. The cost to districts is very minimal and often time free of charge is a KSDE consultant can deliver the training. The training will be customized to the needs of the district. To request a training please go to http://community.ksde.org/Default.aspx?tabid=5812 and complete the training request form.

For questions related to mathematics in Kansas please contact Melissa Fast at mfast@ksde.org.

KATM: Zone 3 Update

Zone 3 Rep: Stacey Bell (staceybell@katm.org)

Kansas Mathematics Standards

Draft

12-28-16

K-12 Draft of KS Math Standards

KSDE released the draft of standards for public review

The draft of standards are out for the public to review. It is important to read through them and do a crosswalk between the current standards and the proposed draft to see what is different. The 8 Mathematical Practices are still front and center along with the critical issues for each grade level. Betsy Wiens posted about this document on our KATM Facebook page. We would love to hear your thoughts about the changes.

Stacey Bell
Serves as the Zone 3 Rep for KATM Members.

KATM Facebook

Join us on Facebook: Kansas Association of Teachers of Mathematics

KATM Twitter

Follow us on Twitter: KATM @KATMWebmaster

1
BE INFORMED
Go to http://community.ksde.org/Default.aspx?tabid=6151

2
DISCUSS ON FACEBOOK
Join the discussion on the KATM Facebook.

3
EMAIL ME
You are more than welcome to email me with questions. I will try to find out answers.

Call for Nominations for KATM

President-elect, Vice Pres. of College and Vice Pres. of Middle School

Are you or someone you know interested in getting involved in KATM? We are looking for educators that are interested in taking a leadership role in the field of Math Education throughout the State of Kansas. You can nominate yourself or someone that you know that has demonstrated a passion for advancing math in our state as well as someone that has a lot to offer in the way of supporting teachers. Please fill out the online nomination form found at katm.org under the nominations tab. Regular members in good standing are eligible for positions on the KATM Board. Nominations need to be completed by the end of January so we can post bios in the Feb. Bulletin. Elections will be held online at www.katm.org in March. A notice will be sent to remind you to vote.

The **President-Elect position** is a 4 year term. You start as President-Elect for one year, President your second year, and then serve as Past President for 2 years. We are also looking

We Need You!

Run For An Office!
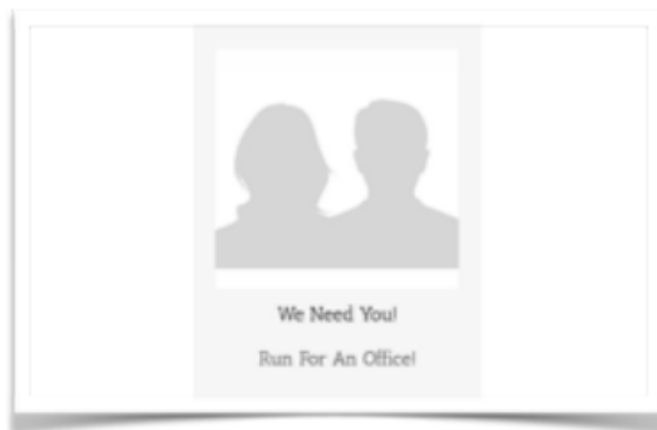
for **Vice President - College** and **Vice President - of Middle School**. VP's promote issues from their level and serve as liaisons for college or middle school teachers.

"We are always looking for people to serve on our KATM Board!"

The KATM Board meets 3 times a year and put on an Annual State Math Conference. Your meals, travel and lodging are covered to attend the Board Meetings and the Conference. We are always looking for people to serve on our KATM Board!

KATM: PROVIDING PROFESSIONAL DEVELOPMENT FOR KANSAS

2017 Conference news: We are working on creating a flyer that includes a graphic and theme for the conference. We are ready to accept conference registrations and putting the final touches on the vendor registrations. We are needing your help! Please consider submitting a proposal to present at the conference on Oct. 16, 2017. You can find the proposal link under the Conference tab in our website: katm.org. Proposals will be accepted from now until August 1. Once your proposal has been approved, you will hear from Amy Johnston, and then you can register. The registration for a speaker is $10.00. Regular registration will be $70.00. Click on the link below the picture to go directly to proposal form.

2017 Conference Presentation Proposal Form

Lead Presenter Name

2017 KATM

Do you like what you find in this Bulletin? Would you like to receive more Bulletins, as well as other benefits?

Consider becoming a member of KATM.

For just $15 a year, you can become a member of KATM and have the Bulletin e-mailed to you as soon as it becomes available. KATM publishes 4 Bulletins a year. Many of the resources from our Bulletin are also available when you lo in to our website. In addition, as a KATM member, you can apply for two different $1000 scholarship.

You can sign up online at katm.org, under the Membership Tab! Join KATM today!

Join us today!!! Complete the form below and send it with your check payable to
KATM to:
Margie Hill
KATM-Membership
15735 Antioch Road
Overland Park, Kansas 66221

Name_____

Address_____

City_____

State_____

Zip_____

Home Phone_____

HOME or PERSONAL EMAIL:

_____

Are you a member of NCTM? Yes____ No____
Position: (Cirlce only one)
       Parent
       Teacher::   Level(s)_____
       Dept. Chair
       Supervisor
       Other

Referred by: _____
_____
KANSAS ASSOCIATION MEMBERSHIPS
Individual Membership: $15/yr. ____
 Three Years: $40 ____
Student Membership: $ 5/yr. ____
Institutional Membership: $25/yr. ____
Retired Teacher Membership: $ 5/yr. ____
First Year Teacher Membership:$5/yr. ____
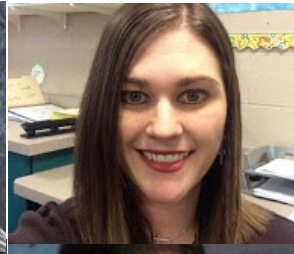Spousal Membership: $ 5/yr. ____
(open to spouses of current members who hold a regular Individual Membership in KATM)

KATM Executive Board Members

President: David Fernkopf, Principal, Overbrook Attendance Center, dferkopf at usd434.us

President: Elect: Stacey Ryan, Middle School teacher, Anodover Middle School, ryans@usd385.org

Past President, NCTM Rep: Stacey Bell, Instructional Coach, Shawnee Heights Middle School

bells at usd450.net, 785-379-5830

Past President, Community Relations: Pat Foster
Principal, Oskaloosa Elementary School
pfoster at usd341.org

Secretary: Janet Stramel, Assistant Professor, Fort Hays State Univ.

jkstramel at fhsu.edu
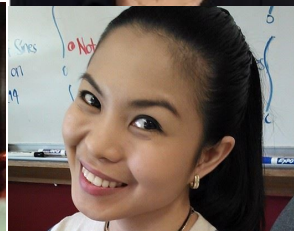
Vice President, College: Lanee Young

Membership Co-chairs: Margie Hill, Instructor, Kansas University

marghill at @ ku.edu

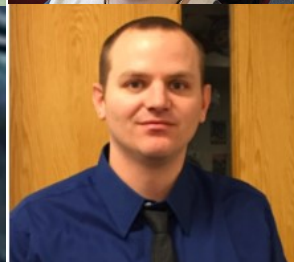**Vice President High School:** Cherryl Delacruz, High Park High School

cdelacruz at tps501.org

Membership Co-Chair: Betsy Wiens, Math Consultant

betsy.wiens at gmail.com

Vice President Middle School: Blake Carlson, Middle School Math Teacher, French Middle School
bcarlson at tps501.org

Treasurer: David Fernkopf, Principal, Overbrook Attendance Center, dferkopf at usd434.us

Vice President Elementary: Amy Johnston, 2nd grade Teacher, Auburn Elementary

johnsamy at usd437.com

KSDE Liaison: Melissa Fast, Math Education Consultant
mfast at ksde.org

Bulletin Editor: Jenny Wilcox, 7th grade teacher, Washburn Rural Middle School,
wilcojen at usd437.net

KATM Executive Board Members

Zone 1 Coordinator:

Jerry Braun, Hays Middle School, jj_ks at yahoo.com

Zone 2 Coordinator:

Kira Pearce

Zone 3 Coordinator:

This position is currently open.

Zone 4 Coordinator:

Lara Staker

Zone 5 Coordinator:

Lisa Lajoie-Smith, Instructional Consultant, llajoie at sped618.org

Zone 6 Coordinator:

Jeanett Moore, 2nd grade teacher, USD 48

Jeanett.moore at usd480.net

Webmaster: Fred Hollingshead Instructional Coach, Shawnee Heights High School hollingsheadf at usd450.net