Welcome KATM!

Improving Mathematics Instruction for Students who Struggle

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Topics for Today

- Who's struggling? Brief overview of **RtI Model**, one version of a multi-tiered system of support (**MTSS**)
- What helps students build **cognitive structures** and **connections** in mathematics? What doesn't help???
- Research based **Interventions** to <u>try</u> (not buy)
- **Strategies** for teaching math that **DON'T EXPIRE!!**

Why aren't Tier 2 Interventions Helping?

- Recent studies reveal that teachers providing Tier 2 mathematics interventions to elementary and middle grade students largely use computational **worksheets** (Foegen & Dougherty, 2010; Swanson, Solis, Ciullo & McKenna, 2012)
- In my travels to classrooms and schools many use a **one-size-fits-all generic drill and kill computer program** (a worksheet on a computer).

Worksheets + computer programs ≠ understanding

Intervention Recommendations from Research

- **C**oncrete--**S**emi-Concrete--**A**bstract (CSA) visual approach
- Explicit instruction
- Underlying mathematical structures

Based on:
Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). A summary of nine key studies: Multi-tier intervention and response to interventions for students struggling in mathematics. Portsmouth, NH: RMC Research Corporation, Center on Instruction.
Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. New York: Routledge.

Function Table – Finding the Rule

In	Out
1	2
2	4
3	6
4	8
5	
20	
n	

Understand that a function is a rule that assigns to each input exactly one output- enhancing algebraic thinking

Van de Walle, J., Karp, K., & Bay Williams, J. (2016). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

So, What did You Learn in School?

- With the person sitting next to or around you, discuss these rules – were you taught them in elementary school?
- Decide if the rules shown at the right are always true.
- If the rule is not always true, find a counterexample.

- Addition makes numbers bigger
- Multiplication makes numbers bigger.
- When you multiply a number by 10, just put a 0 on the end of the number.
- PEMDAS.

Addition and Multiplication make "Bigger"

32 + 67 = 99

– 3 + (–14) = –17

–17 is neither larger than –3 nor –14.

15 + 0 = 15

15 x 10 = 150

$\frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$

0.25 x 0.16 = 0.04

15 x 0 = 0

When you multiply by 10, just put a 0 on the end of the number.

15 x 10 = 150

4.5 x 10 = 45.0

4.5 x 10 ≠ 4.50

Impact of Teaching Rules that Expire

- Students use rules as they have interpreted them.
- They often do not think about the rule beyond its immediate application.
- When even the strongest math students find that a "rule" doesn't work, it is unnerving and scary.

Take the Oath!! Nevermore:

- Borrowing
- Carrying
- "Reducing" fractions
- Talking about Fractions as a "Top Number" and a "Bottom Number"
- "Plugging" numbers into the equation
- Getting "rid" of the decimal
- Diagonal fraction bar

What do we know?

- Telling isn't teaching.
- Told isn't taught.
- Interventions provide opportunities to spend time actively developing concepts and mathematical structure.

Active Mathematics:
Boaler, J. & Selling, S.K. (2017) Psychological imprisonment or intellectual freedom? A longitudinal study of contrasting school mathematics approaches and their impact on adults' lives. *Journal for Research in Mathematics Education* 48 (1), 78-105.

Let's start with Word Problems

At all grades students who struggle see each problem as a **separate endeavor**

They **focus on steps** to follow rather than the behavior of the operations

They tend to use **trial and error** – (disconnected thinking – not relational thinking)

They need to focus on **actions**, **representations** and **general properties of the operations**

Wynn has 9 cookies. She wants to give these cookies in equal amounts to 3 friends. How many cookies will each friend receive?

9 ÷ 3 = ? Group size unknown

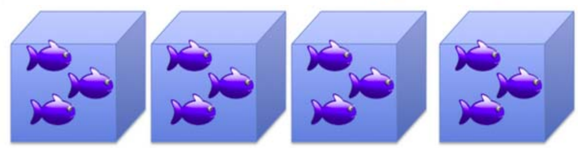
Sam wants to put 15 cookies on plates with 5 on each. How many plates will he need?

15 ÷ 5 = ? Number of groups unknown

CCSS Appendix – Common Multiplication and Division Situations

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

So, we are still not sure our students can handle this…



Suppose there are 4 tanks and 3 fish in each tank. The total number of fish in this situation can be expressed as $4 \times 3 = 12$.

a. Describe what is meant in this situation by $12 \div 3 = 4$

b. Describe what is meant in this situation by $12 \div 4 = 3$

Can an Intervention Provide time to Discuss Options?

How could students talk about which of the following three options would be the correct answer?

- The shepherd is 30 years old
- The shepherd is 125 years old; and
- It is not possible to tell the shepherd's age from the information given in the problem.

Caldwell, Kobett & Karp (2014) *Essential understanding of addition and subtraction in practice, grades K-2*. NCTM.

Why Avoid a Key Word Strategy?

- The use of a Key Word Strategy does not—
  - Develop of sense making or support making meaning
  - Build structures for more advanced learning
  - Appear in many problems
- Students consistently use Key Words inappropriately
- Multi-step problems are impossible to solve with a Key Word Strategy (and two step problems start in 2nd grade)

Clement & Bernhard, 2005 A problem solving alternative to using key words.
Van de Walle, J., Karp, K., & Bay Williams, J. (2016). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

What is the Whole School Agreement?

- Decide on the language and models everyone will use – focusing on precision and consistency
- Prepare all students, from the beginning, to walk out of the building with the mathematical knowledge and processes they need
- Engage each and every student in “doing mathematics” to build long lasting understanding

Cai, J. (2010). Helping elementary school students become successful mathematical problem solvers. In D. Lambdin (Ed.), *Teaching and learning mathematics: Translating research to the classroom* (pp. 9–14). Reston, VA: NCTM.
Karp, Bush & Dougherty (2016) *Establishing a Mathematics Whole School Agreement*. NCTM.
Stein, Smith, Henningsen & Silver, 2000 - Mathematical Tasks Framework

When you Return to your School

1. What are the models your school can agree to use?
2. What is the language that you agree to use? What language should be avoided?
3. What notations should be used? Must be avoided?
4. What is an example of a concept or model moving vertically up the grades?

Recap – What Should be Emphasized in Interventions

❖Action and the importance of “doing mathematics”

❖By having students carry out the actions – mental residue results!!

❖Use intervention sessions as opportunities to make math MEMORABLE

Mental Residue - Dougherty, B. J. (2008). Measure up: A quantitative view of early algebra. In Kaput, J. J.,b, D. W., & Blanton, M. L. (Eds.), *Algebra in the early grades*, (pp. 389–412). Mahwah, NJ: Erlbaum.

References and Contact Info

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